Practice Problem 10.16

In the Wien-bridge oscillator circuit in Fig. 10.42, let $R_1 = R_2 = 2.5\ \text{k}\Omega$, $C_1 = C_2 = 1$ nF. Determine the frequency f_o of the oscillator.

Answer: 63.66 kHz.

10.10 Summary

1. We apply nodal and mesh analysis to ac circuits by applying KCL and KVL to the phasor form of the circuits.
2. In solving for the steady-state response of a circuit that has independent sources with different frequencies, each independent source *must* be considered separately. The most natural approach to analyzing such circuits is to apply the superposition theorem. A separate phasor circuit for each frequency *must* be solved independently, and the corresponding response should be obtained in the time domain. The overall response is the sum of the time domain responses of all the individual phasor circuits.
3. The concept of source transformation is also applicable in the frequency domain.
4. The Thevenin equivalent of an ac circuit consists of a voltage source $\mathbf{V}_{\text{Th}}$ in series with the Thevenin impedance $\mathbf{Z}_{\text{Th}}$.
5. The Norton equivalent of an ac circuit consists of a current source $\mathbf{I}_N$ in parallel with the Norton impedance $\mathbf{Z}_N\,(=\mathbf{Z}_{\text{Th}})$.
6. *PSpice* is a simple and powerful tool for solving ac circuit problems. It relieves us of the tedious task of working with the complex numbers involved in steady-state analysis.
7. The capacitance multiplier and the ac oscillator provide two typical applications for the concepts presented in this chapter. A capacitance multiplier is an op amp circuit used in producing a multiple of a physical capacitance. An oscillator is a device that uses a dc input to generate an ac output.

Review Questions

10.1 The voltage $\mathbf{V}_o$ across the capacitor in Fig. 10.43 is:

(a) $5\angle 0^\circ$ V (b) $7.071\angle 45^\circ$ V
(c) $7.071\angle -45^\circ$ V (d) $5\angle -45^\circ$ V

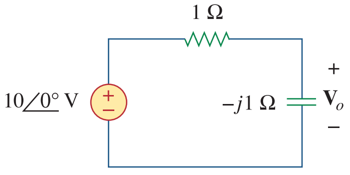

Figure 10.43
For Review Question 10.1.

10.2 The value of the current $\mathbf{I}_o$ in the circuit of Fig. 10.44 is:

(a) $4\angle 0^\circ$ A (b) $2.4\angle -90^\circ$ A
(c) $0.6\angle 0^\circ$ A (d) -1 A

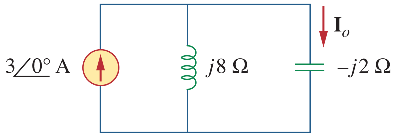

Figure 10.44
For Review Question 10.2.

10.3 Using nodal analysis, the value of $\mathbf{V}_o$ in the circuit of Fig. 10.45 is:

(a) −24 V (b) −8 V
(c) 8 V (d) 24 V

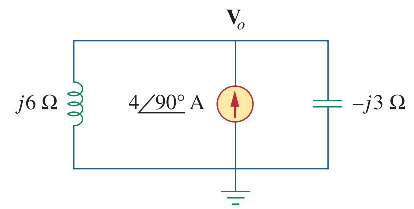

Figure 10.45
For Review Question 10.3.

10.4 In the circuit of Fig. 10.46, current $i(t)$ is:

(a) 10 cos t A (b) 10 sin t A (c) 5 cos t A
(d) 5 sin t A (e) 4.472 cos(t − 63.43°) A

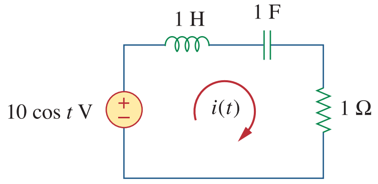

Figure 10.46
For Review Question 10.4.

10.5 Refer to the circuit in Fig. 10.47 and observe that the two sources do not have the same frequency. The current $i_x(t)$ can be obtained by:

(a) source transformation
(b) the superposition theorem
(c) *PSpice*

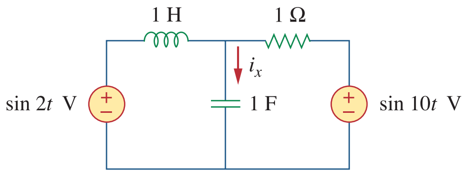

Figure 10.47
For Review Question 10.5.

10.6 For the circuit in Fig. 10.48, the Thevenin impedance at terminals a-b is:

(a) 1 Ω (b) 0.5 − j0.5 Ω
(c) 0.5 + j0.5 Ω (d) 1 + j2 Ω
(e) 1 − j2 Ω

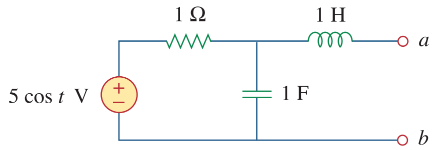

Figure 10.48
For Review Questions 10.6 and 10.7.

10.7 In the circuit of Fig. 10.48, the Thevenin voltage at terminals a-b is:

(a) $3.535/-45°$ V (b) $3.535/45°$ V
(c) $7.071/-45°$ V (d) $7.071/45°$ V

10.8 Refer to the circuit in Fig. 10.49. The Norton equivalent impedance at terminals a-b is:

(a) $-j4\ \Omega$ (b) $-j2\ \Omega$
(c) $j2\ \Omega$ (d) $j4\ \Omega$

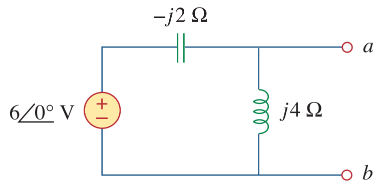

Figure 10.49
For Review Questions 10.8 and 10.9.

10.9 The Norton current at terminals a-b in the circuit of Fig. 10.49 is:

(a) $1/0°$ A (b) $1.5/-90°$ A
(c) $1.5/90°$ A (d) $3/90°$ A

10.10 *PSpice* can handle a circuit with two independent sources of different frequencies.

(a) True (b) False

Answers: 10.1c, 10.2a, 10.3d, 10.4a, 10.5b, 10.6c, 10.7a, 10.8a, 10.9d, 10.10b.

Problems

Section 10.2 Nodal Analysis

10.1 Determine i in the circuit of Fig. 10.50.

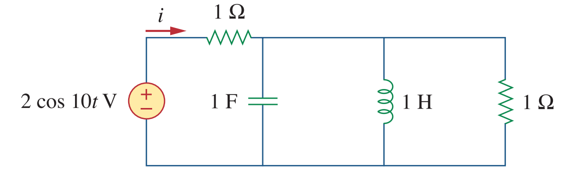

Figure 10.50
For Prob. 10.1.

10.2 Using Fig. 10.51, design a problem to help other students better understand nodal analysis.

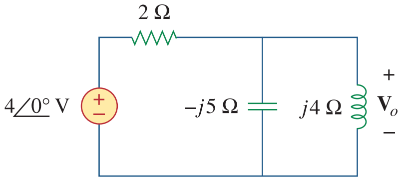

Figure 10.51
For Prob. 10.2.

10.3 Determine v_o in the circuit of Fig. 10.52.

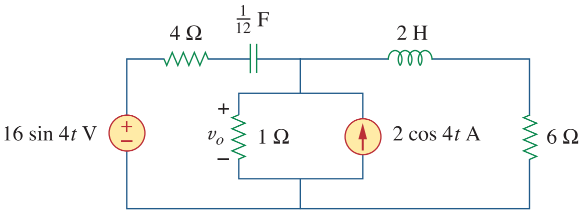

Figure 10.52
For Prob. 10.3.

10.4 Compute $v_o(t)$ in the circuit of Fig. 10.53.

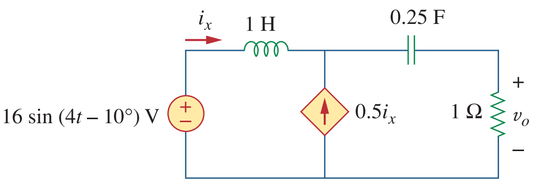

Figure 10.53
For Prob. 10.4.

10.5 Find i_o in the circuit of Fig. 10.54.

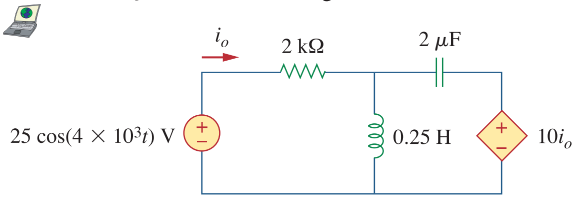

Figure 10.54
For Prob. 10.5.

10.6 Determine $\mathbf{V}_x$ in Fig. 10.55.

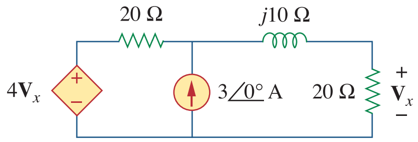

Figure 10.55
For Prob. 10.6.

10.7 Use nodal analysis to find $\mathbf{V}$ in the circuit of Fig. 10.56.

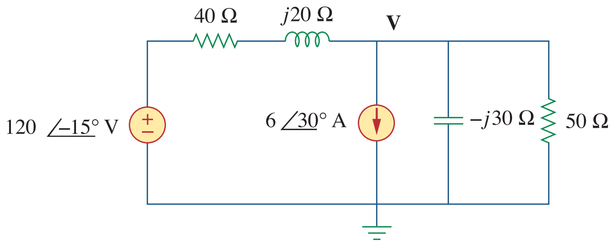

Figure 10.56
For Prob. 10.7.

10.8 Use nodal analysis to find current i_o in the circuit of Fig. 10.57. Let $i_s = 6 \cos(200t + 15°)$ A.

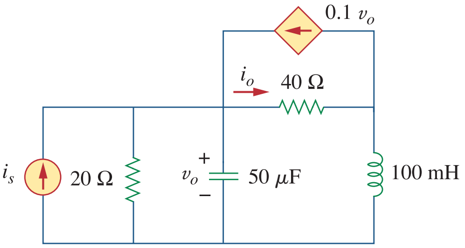

Figure 10.57
For Prob. 10.8.

10.9 Use nodal analysis to find v_o in the circuit of Fig. 10.58.

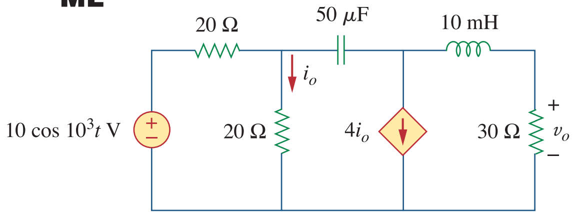

Figure 10.58
For Prob. 10.9.

10.10 Use nodal analysis to find v_o in the circuit of Fig. 10.59. Let $\omega = 2$ krad/s.

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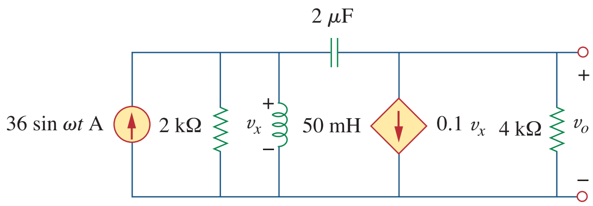

Figure 10.59
For Prob. 10.10.

10.11 Using nodal analysis, find $i_o(t)$ in the circuit in Fig. 10.60.

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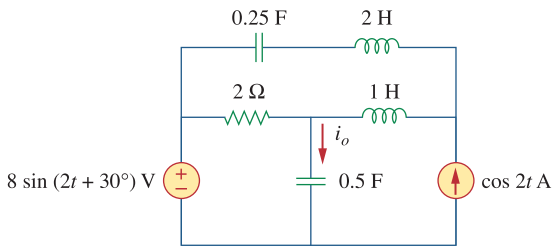

Figure 10.60
For Prob. 10.11.

10.12 Using Fig. 10.61, design a problem to help other students better understand nodal analysis.

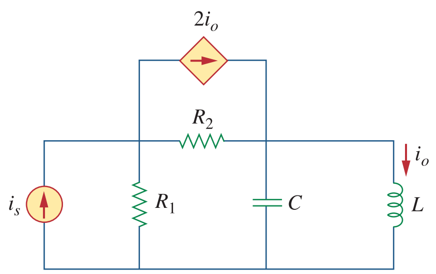

Figure 10.61
For Prob. 10.12.

10.13 Determine $\mathbf{V}_x$ in the circuit of Fig. 10.62 using any method of your choice.

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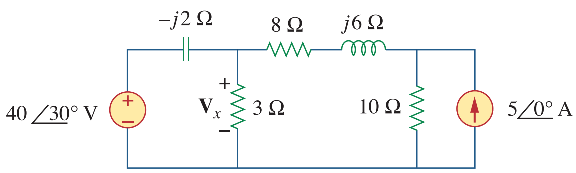

Figure 10.62
For Prob. 10.13.

10.14 Calculate the voltage at nodes 1 and 2 in the circuit of Fig. 10.63 using nodal analysis.

ML

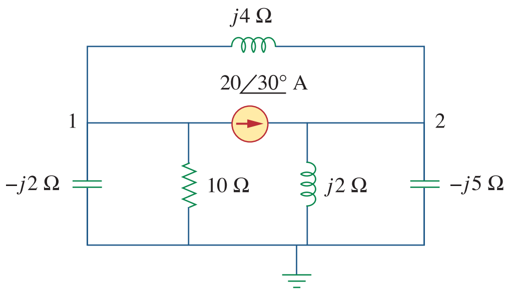

Figure 10.63
For Prob. 10.14.

10.15 Solve for the current **I** in the circuit of Fig. 10.64 using nodal analysis.

ML

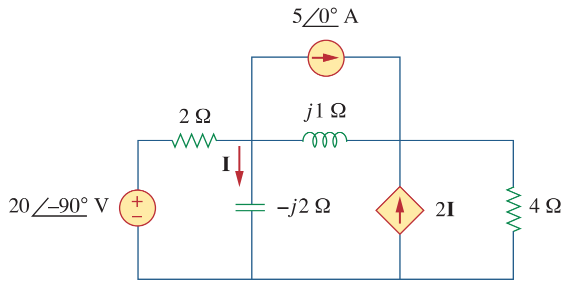

Figure 10.64
For Prob. 10.15.

10.16 Use nodal analysis to find $\mathbf{V}_x$ in the circuit shown in Fig. 10.65.

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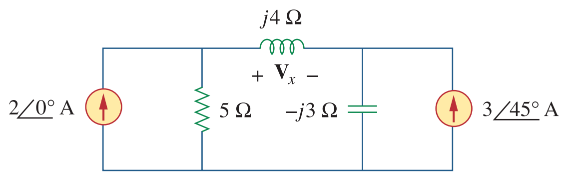

Figure 10.65
For Prob. 10.16.

10.17 By nodal analysis, obtain current $\mathbf{I}_o$ in the circuit of Fig. 10.66.

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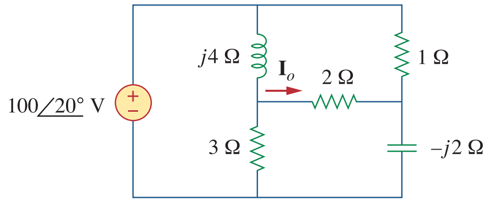

Figure 10.66
For Prob. 10.17.

10.18 Use nodal analysis to obtain $\mathbf{V}_o$ in the circuit of Fig. 10.67 below.

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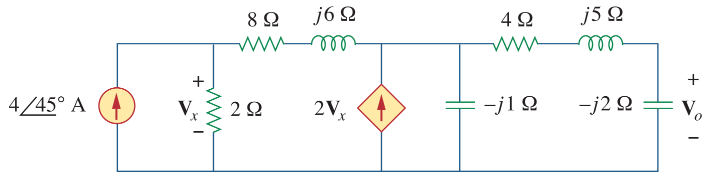

Figure 10.67
For Prob. 10.18.

10.19 Obtain $\mathbf{V}_o$ in Fig. 10.68 using nodal analysis.

ML

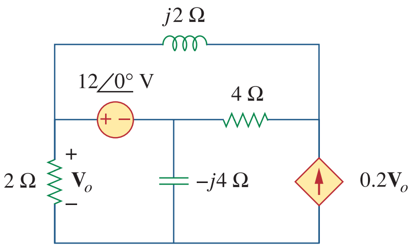

Figure 10.68
For Prob. 10.19.

10.20 Refer to Fig. 10.69. If $v_s(t) = V_m \sin \omega t$ and $v_o(t) = A \sin(\omega t + \phi)$, derive the expressions for A and ϕ.

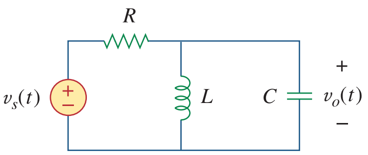

Figure 10.69
For Prob. 10.20.

10.21 For each of the circuits in Fig. 10.70, find $\mathbf{V}_o/\mathbf{V}_i$ for $\omega = 0$, $\omega \to \infty$, and $\omega^2 = 1/LC$.

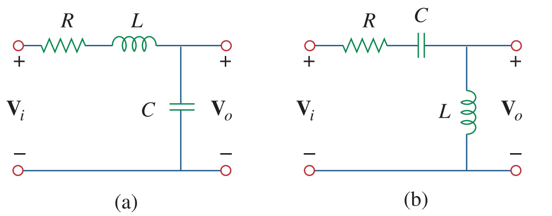

Figure 10.70
For Prob. 10.21.

10.22 For the circuit in Fig. 10.71, determine $\mathbf{V}_o/\mathbf{V}_s$.

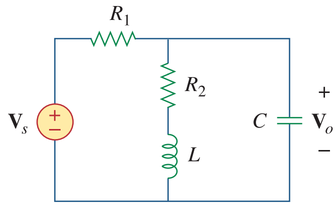

Figure 10.71
For Prob. 10.22.

10.23 Using nodal analysis obtain $\mathbf{V}$ in the circuit of Fig. 10.72.

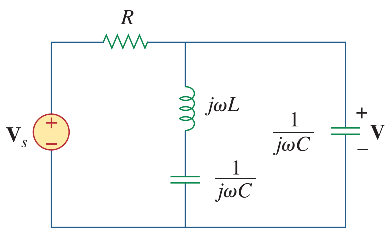

Figure 10.72
For Prob. 10.23.

Section 10.3 Mesh Analysis

10.24 Design a problem to help other students better understand mesh analysis.

10.25 Solve for i_o in Fig. 10.73 using mesh analysis.

ML

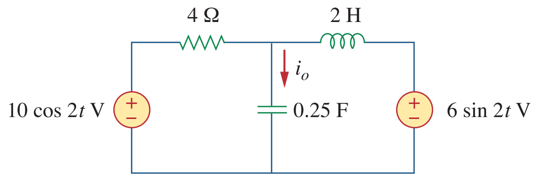

Figure 10.73
For Prob. 10.25.

10.26 Use mesh analysis to find current i_o in the circuit of Fig. 10.74.

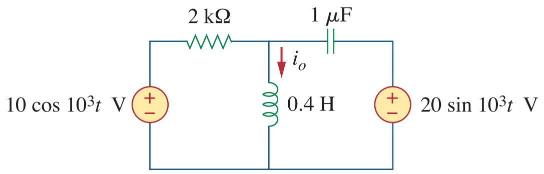

Figure 10.74
For Prob. 10.26.

10.27 Using mesh analysis, find $\mathbf{I}_1$ and $\mathbf{I}_2$ in the circuit of Fig. 10.75.

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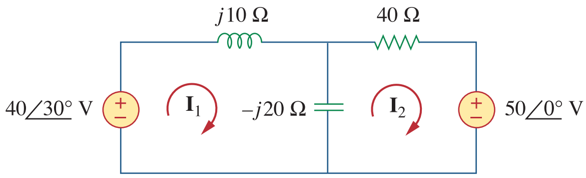

Figure 10.75
For Prob. 10.27.

10.28 In the circuit of Fig. 10.76, determine the mesh currents i_1 and i_2. Let $v_1 = 10 \cos 4t$ **V** and $v_2 = 20 \cos(4t - 30^\circ)$ V.

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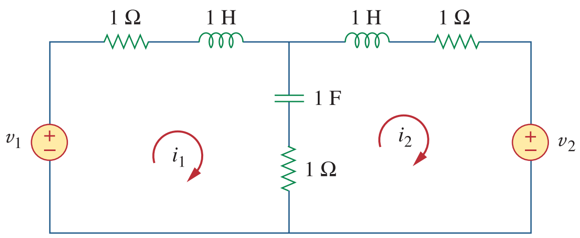

Figure 10.76
For Prob. 10.28.

10.29 Using Fig. 10.77, design a problem to help other students better understand mesh analysis.

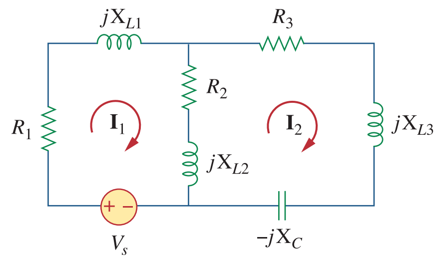

Figure 10.77
For Prob. 10.29.

10.30 Use mesh analysis to find v_o in the circuit of Fig. 10.78. Let $v_{s1} = 120 \cos(100t + 90^\circ)$ V, $v_{s2} = 80 \cos 100t$ V.

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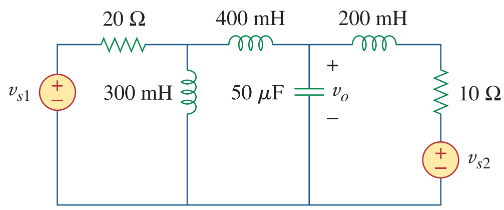

Figure 10.78
For Prob. 10.30.

10.31 Use mesh analysis to determine current $\mathbf{I}_o$ in the circuit of Fig. 10.79 below.

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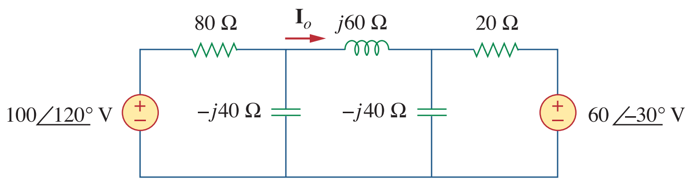

Figure 10.79
For Prob. 10.31.

10.32 Determine $\mathbf{V}_o$ and $\mathbf{I}_o$ in the circuit of Fig. 10.80 using mesh analysis.

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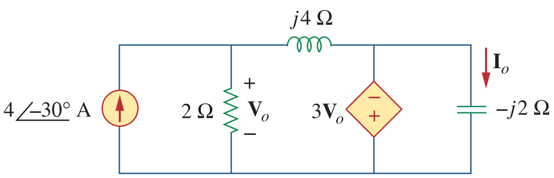

Figure 10.80
For Prob. 10.32.

10.33 Compute **I** in Prob. 10.15 using mesh analysis.

ML

10.34 Use mesh analysis to find $\mathbf{I}_o$ in Fig. 10.28 (for Example 10.10).

ML

10.35 Calculate $\mathbf{I}_o$ in Fig. 10.30 (for Practice Prob. 10.10) using mesh analysis.

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10.36 Compute $\mathbf{V}_o$ in the circuit of Fig. 10.81 using mesh analysis.

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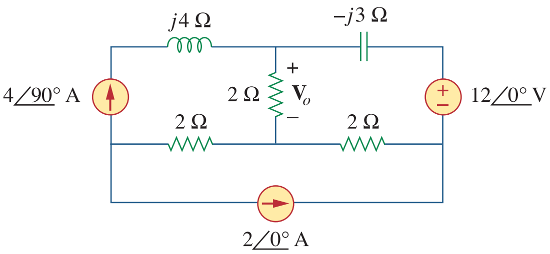

Figure 10.81
For Prob. 10.36.

10.37 Use mesh analysis to find currents $\mathbf{I}_1$, $\mathbf{I}_2$, and $\mathbf{I}_3$ in the circuit of Fig. 10.82.

ML

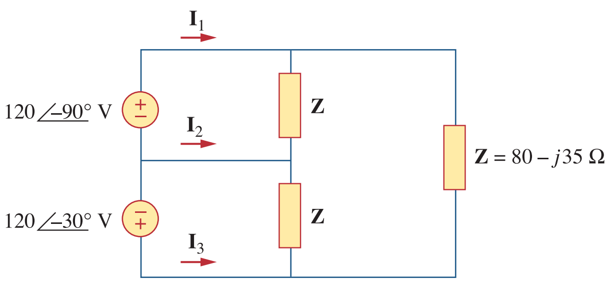

Figure 10.82
For Prob. 10.37.

10.38 Using mesh analysis, obtain $\mathbf{I}_o$ in the circuit shown in Fig. 10.83.

ML

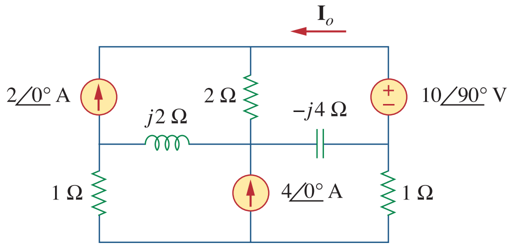

Figure 10.83
For Prob. 10.38.

10.39 Find $\mathbf{I}_1$, $\mathbf{I}_2$, $\mathbf{I}_3$, and $\mathbf{I}_x$ in the circuit of Fig. 10.84.

ML

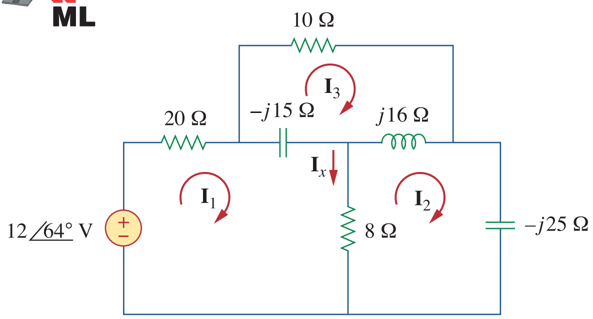

Figure 10.84
For Prob. 10.39.

Section 10.4 Superposition Theorem

10.40 Find i_o in the circuit shown in Fig. 10.85 using superposition.

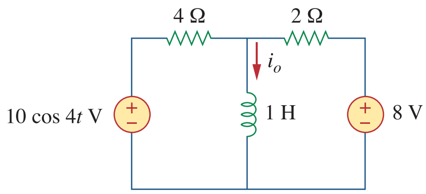

Figure 10.85
For Prob. 10.40.

10.41 Find v_o for the circuit in Fig. 10.86, assuming that $v_s = 6 \cos 2t + 4 \sin 4t$ V.

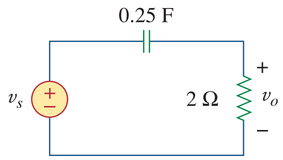

Figure 10.86
For Prob. 10.41.

10.42 Using Fig. 10.87, design a problem to help other students better understand the superposition theorem.

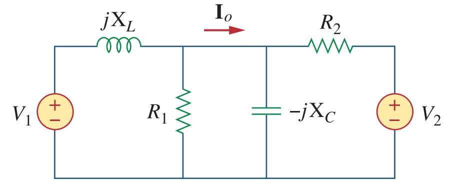

Figure 10.87
For Prob. 10.42.

10.43 Using the superposition principle, find i_x in the circuit of Fig. 10.88.

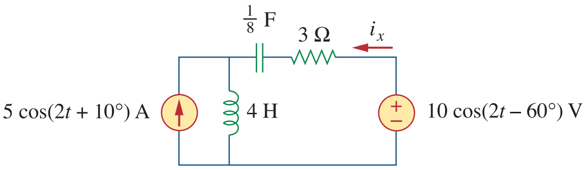

Figure 10.88
For Prob. 10.43.

10.44 Use the superposition principle to obtain v_x in the circuit of Fig. 10.89. Let $v_s = 50 \sin 2t$ V and $i_s = 12 \cos(6t + 10°)$ A.

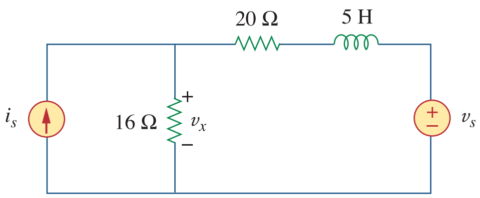

Figure 10.89
For Prob. 10.44.

10.45 Use superposition to find $i(t)$ in the circuit of Fig. 10.90.

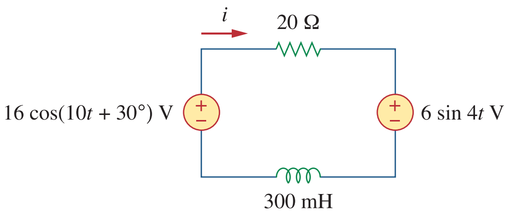

Figure 10.90
For Prob. 10.45.

10.46 Solve for $v_o(t)$ in the circuit of Fig. 10.91 using the superposition principle.

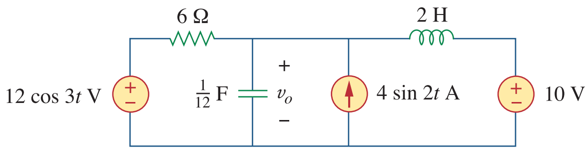

Figure 10.91
For Prob. 10.46.

10.47 Determine i_o in the circuit of Fig. 10.92, using the superposition principle.

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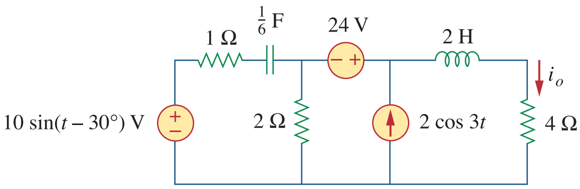

Figure 10.92
For Prob. 10.47.

10.48 Find i_o in the circuit of Fig. 10.93 using superposition.

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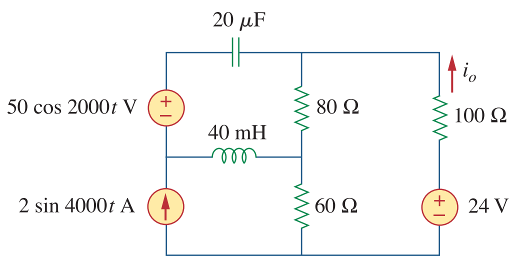


Figure 10.93
For Prob. 10.48.

Section 10.5 Source Transformation

10.49 Using source transformation, find i in the circuit of Fig. 10.94.

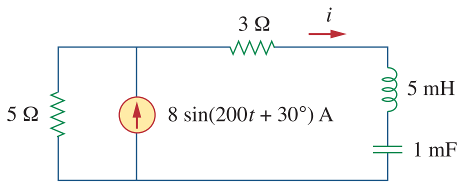


Figure 10.94
For Prob. 10.49.

10.50 Using Fig. 10.95, design a problem to help other students understand source transformation.

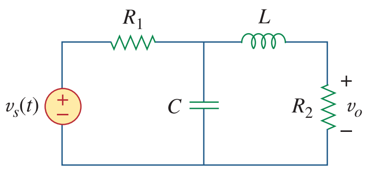


Figure 10.95
For Prob. 10.50.

10.51 Use source transformation to find $\mathbf{I}_o$ in the circuit of Prob. 10.42.

10.52 Use the method of source transformation to find $\mathbf{I}_x$ in the circuit of Fig. 10.96.

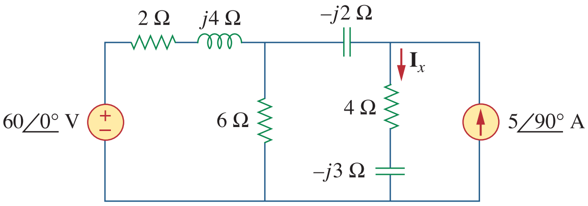


Figure 10.96
For Prob. 10.52.

10.53 Use the concept of source transformation to find $\mathbf{V}_o$ in the circuit of Fig. 10.97.

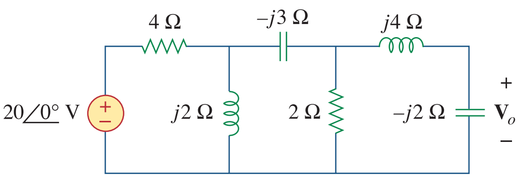
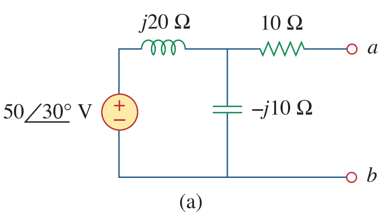


Figure 10.97
For Prob. 10.53.

10.54 Rework Prob. 10.7 using source transformation.

Section 10.6 Thevenin and Norton Equivalent Circuits

10.55 Find the Thevenin and Norton equivalent circuits at terminals *a-b* for each of the circuits in Fig. 10.98.



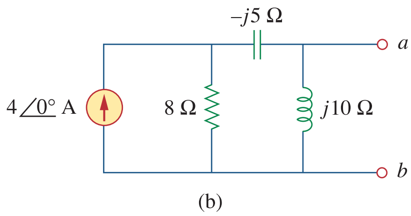


Figure 10.98
For Prob. 10.55.

10.56 For each of the circuits in Fig. 10.99, obtain Thevenin and Norton equivalent circuits at terminals *a-b*.

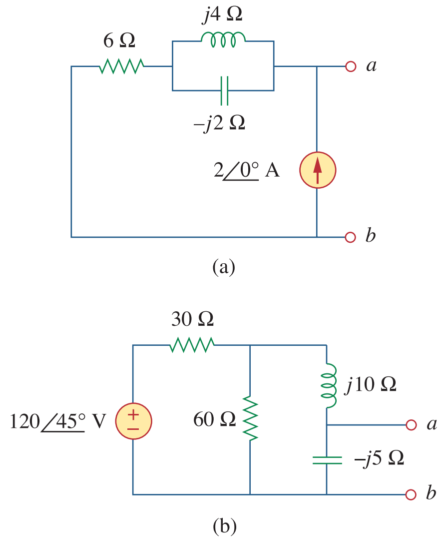


Figure 10.99
For Prob. 10.56.

10.57 Using Fig. 10.100, design a problem to help other students better understand Thevenin and Norton equivalent circuits.

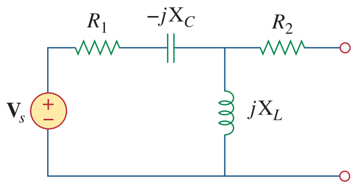


Figure 10.100
For Prob. 10.57.

10.58 For the circuit depicted in Fig. 10.101, find the Thevenin equivalent circuit at terminals *a-b*.

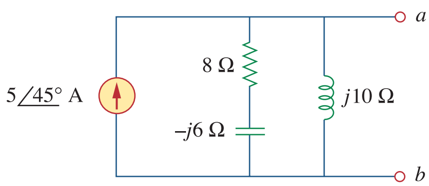


Figure 10.101
For Prob. 10.58.

10.59 Calculate the output impedance of the circuit shown in Fig. 10.102.

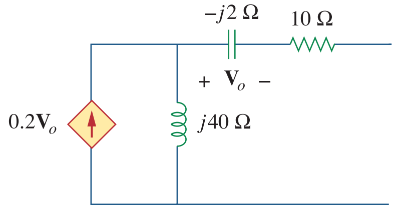


Figure 10.102
For Prob. 10.59.

10.60 Find the Thevenin equivalent of the circuit in Fig. 10.103 as seen from:

(a) terminals *a-b* (b) terminals *c-d*

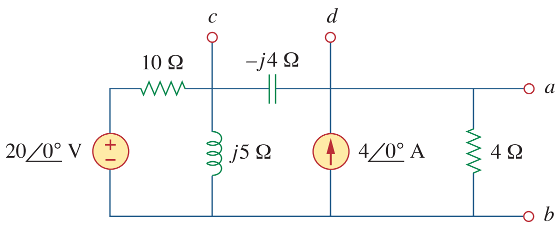


Figure 10.103
For Prob. 10.60.

10.61 Find the Thevenin equivalent at terminals *a-b* of the circuit in Fig. 10.104.

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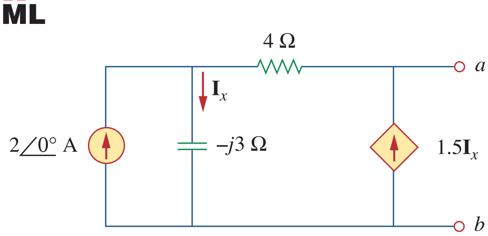

Figure 10.104
For Prob. 10.61.

10.62 Using Thevenin's theorem, find v_o in the circuit of Fig. 10.105.

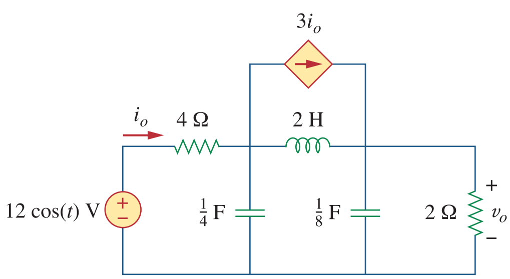

Figure 10.105
For Prob. 10.62.

10.63 Obtain the Norton equivalent of the circuit depicted in Fig. 10.106 at terminals *a-b*.

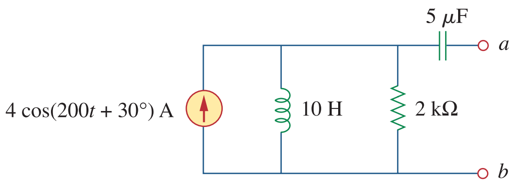

Figure 10.106
For Prob. 10.63.

10.64 For the circuit shown in Fig. 10.107, find the Norton equivalent circuit at terminals *a-b*.

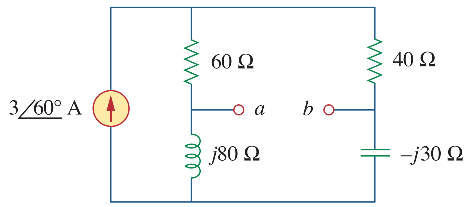

Figure 10.107
For Prob. 10.64.

10.65 Using Fig. 10.108, design a problem to help other students better understand Norton's theorem.

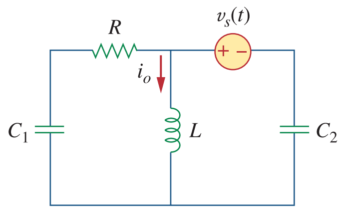

Figure 10.108
For Prob. 10.65.

10.66 At terminals *a-b*, obtain Thevenin and Norton equivalent circuits for the network depicted in Fig. 10.109. Take $\omega = 10$ rad/s.

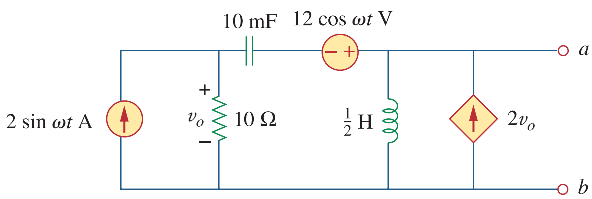

Figure 10.109
For Prob. 10.66.

10.67 Find the Thevenin and Norton equivalent circuits at terminals *a*-*b* in the circuit of Fig. 10.110.

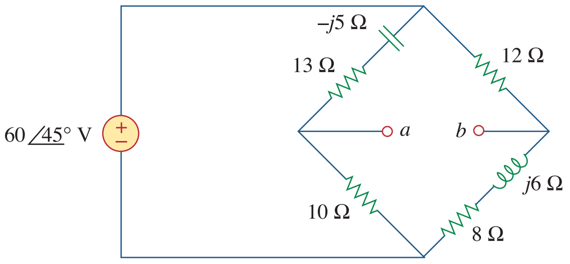

Figure 10.110
For Prob. 10.67.

10.68 Find the Thevenin equivalent at terminals *a*-*b* in the circuit of Fig. 10.111.

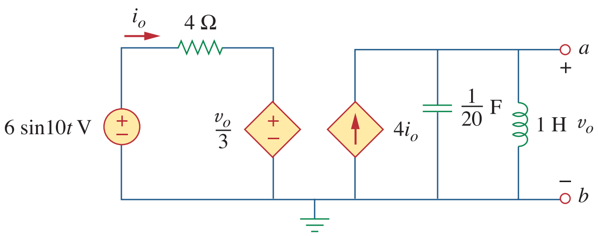

Figure 10.111
For Prob. 10.68.

Section 10.7 Op Amp AC Circuits

10.69 For the differentiator shown in Fig. 10.112, obtain $\mathbf{V}_o/\mathbf{V}_s$. Find $v_o(t)$ when $v_s(t) = \mathbf{V}_m \sin \omega t$ and $\omega = 1/RC$.

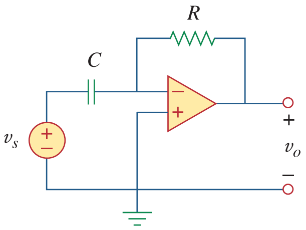

Figure 10.112
For Prob. 10.69.

10.70 Using Fig. 10.113, design a problem to help other students better understand op amps in AC circuits.

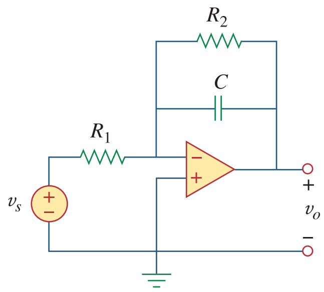

Figure 10.113
For Prob. 10.70.

10.71 Find v_o in the op amp circuit of Fig. 10.114.

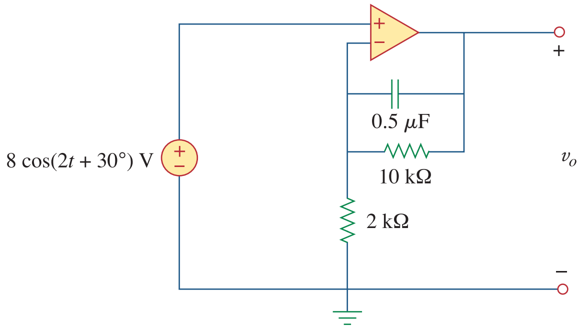

Figure 10.114
For Prob. 10.71.

10.72 Compute $i_o(t)$ in the op amp circuit in Fig. 10.115 if $v_s = 4\cos(10^4 t)$ V.

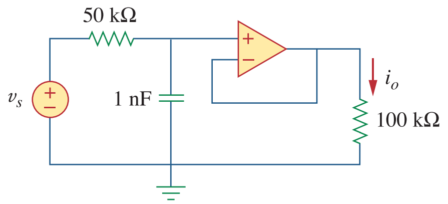

Figure 10.115
For Prob. 10.72.

10.73 If the input impedance is defined as $\mathbf{Z}_{\text{in}} = \mathbf{V}_s/\mathbf{I}_s$, find the input impedance of the op amp circuit in Fig. 10.116 when $R_1 = 10$ kΩ, $R_2 = 20$ kΩ, $C_1 = 10$ nF, $C_2 = 20$ nF, and $\omega = 5000$ rad/s.

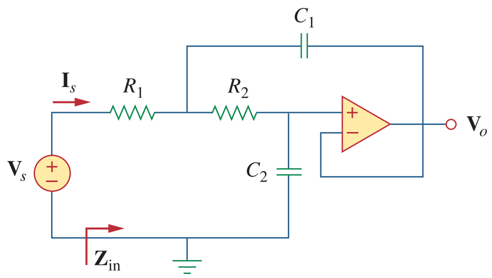

Figure 10.116
For Prob. 10.73.

10.74 Evaluate the voltage gain $\mathbf{A}_v = \mathbf{V}_o/\mathbf{V}_s$ in the op amp circuit of Fig. 10.117. Find $\mathbf{A}_v$ at $\omega = 0$, $\omega \to \infty$, $\omega = 1/R_1C_1$, and $\omega = 1/R_2C_2$.

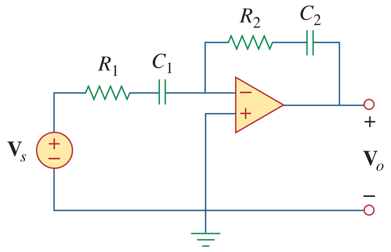

Figure 10.117
For Prob. 10.74.

10.75 In the op amp circuit of Fig. 10.118, find the closed-loop gain and phase shift of the output voltage with respect to the input voltage if $C_1 = C_2 = 1$ nF, $R_1 = R_2 = 100$ kΩ, $R_3 = 20$ kΩ, $R_4 = 40$ kΩ, and $\omega = 2000$ rad/s.

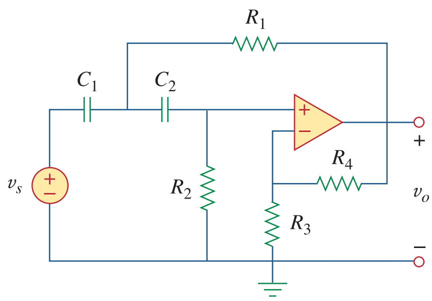

Figure 10.118
For Prob. 10.75.

10.76 Determine $\mathbf{V}_o$ and $\mathbf{I}_o$ in the op amp circuit of Fig. 10.119.

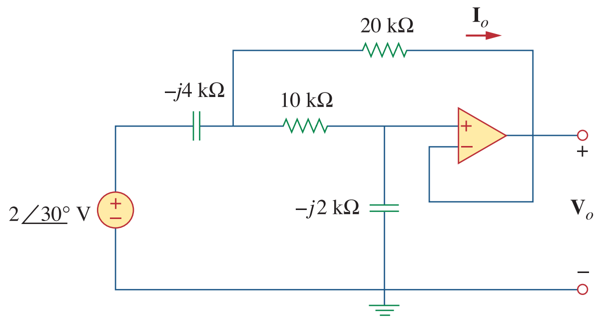

Figure 10.119
For Prob. 10.76.

10.77 Compute the closed-loop gain $\mathbf{V}_o/\mathbf{V}_s$ for the op amp circuit of Fig. 10.120.

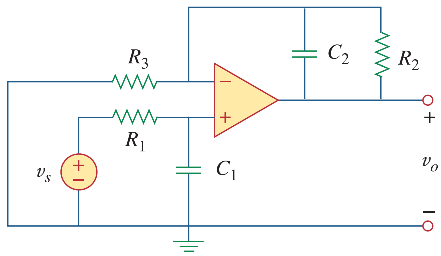

Figure 10.120
For Prob. 10.77.

10.78 Determine $v_o(t)$ in the op amp circuit in Fig. 10.121 below.

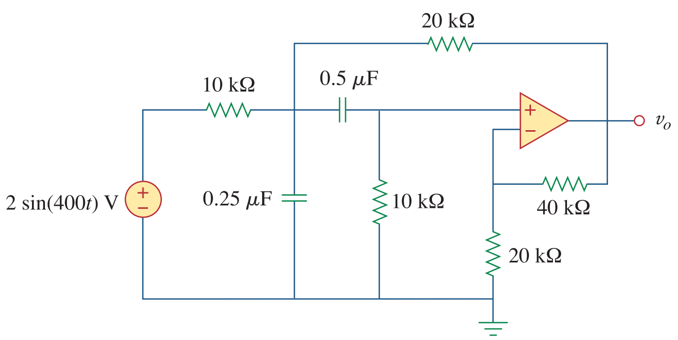

Figure 10.121
For Prob. 10.78.

10.79 For the op amp circuit in Fig. 10.122, obtain $v_o(t)$.

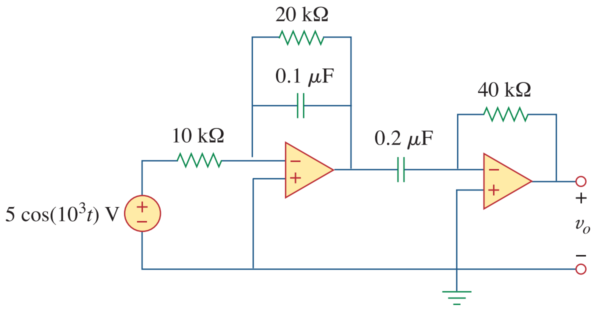

Figure 10.122
For Prob. 10.79.

10.80 Obtain $v_o(t)$ for the op amp circuit in Fig. 10.123 if $v_s = 4\cos(1000t - 60°)$ V.

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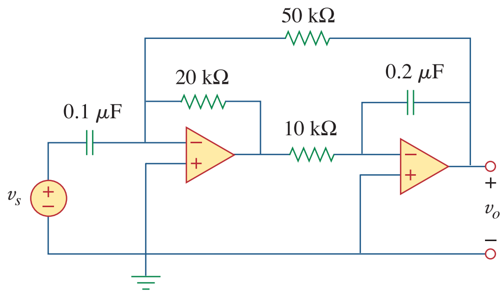


Figure 10.123
For Prob. 10.80.

Section 10.8 AC Analysis Using *PSpice*

10.81 Use *PSpice or MultiSim* to determine $\mathbf{V}_o$ in the circuit of Fig. 10.124. Assume $\omega = 1$ rad/s.

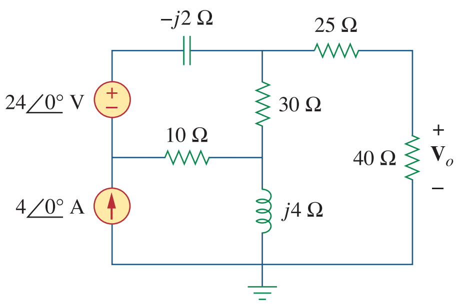


Figure 10.124
For Prob. 10.81.

10.82 Solve Prob. 10.19 using *PSpice or MultiSim*.

10.83 Use *PSpice or MultiSim* to find $v_o(t)$ in the circuit of Fig. 10.125. Let $i_s = 2\cos(10^3 t)$ A.

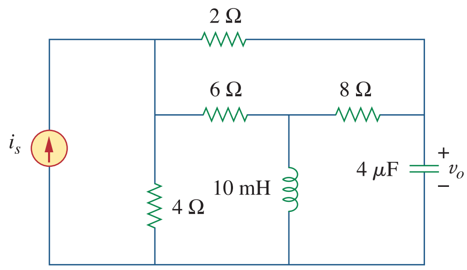


Figure 10.125
For Prob. 10.83.

10.84 Obtain $\mathbf{V}_o$ in the circuit of Fig. 10.126 using *PSpice or MultiSim*.

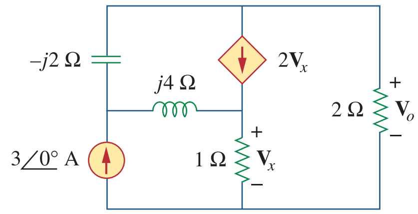


Figure 10.126
For Prob. 10.84.

10.85 Using Fig. 10.127, design a problem to help other students better understand performing AC analysis with *PSpice or MultiSim*.

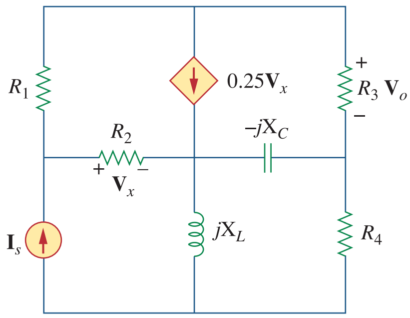


Figure 10.127
For Prob. 10.85.

10.86 Use *PSpice or MultiSim* to find $\mathbf{V}_1$, $\mathbf{V}_2$, and $\mathbf{V}_3$ in the network of Fig. 10.128.

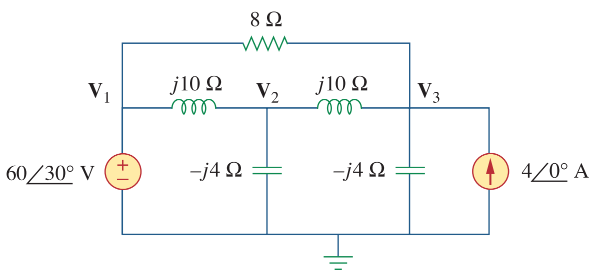


Figure 10.128
For Prob. 10.86.

10.87 Determine $\mathbf{V}_1$, $\mathbf{V}_2$, and $\mathbf{V}_3$ in the circuit of Fig. 10.129 using *PSpice or MultiSim.*

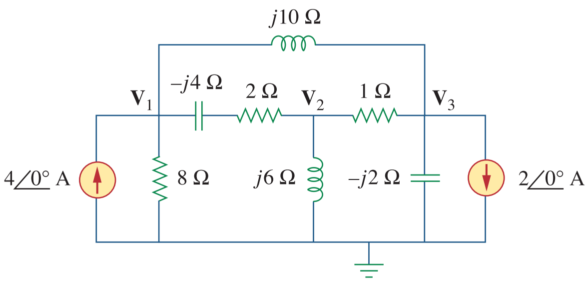


Figure 10.129
For Prob. 10.87.

10.88 Use *PSpice or MultiSim* to find v_o and i_o in the circuit of Fig. 10.130 below.

10.90 Figure 10.132 shows a Wien-bridge network. Show that the frequency at which the phase shift between the input and output signals is zero is $f = \frac{1}{2}\pi RC$, and that the necessary gain is $\mathbf{A}_v = \mathbf{V}_o/\mathbf{V}_i = 3$ at that frequency.

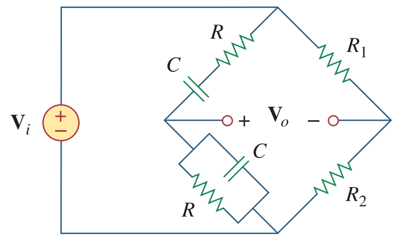


Figure 10.132
For Prob. 10.90.

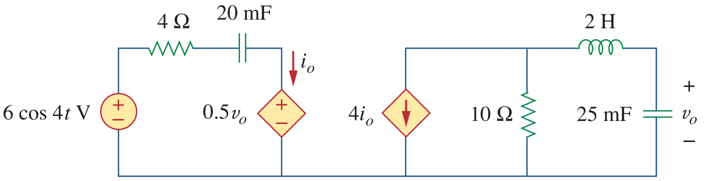


Figure 10.130
For Prob. 10.88.

Section 10.9 Applications

10.89 The op amp circuit in Fig. 10.131 is called an *inductance simulator*. Show that the input impedance is given by

$$\mathbf{Z}_{\text{in}} = \frac{\mathbf{V}_{\text{in}}}{\mathbf{I}_{\text{in}}} = j\omega L_{\text{eq}}$$

where

$$L_{\text{eq}} = \frac{R_1 R_3 R_4}{R_2} C$$

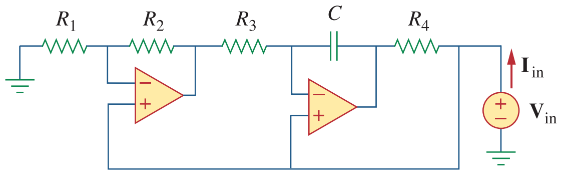


Figure 10.131
For Prob. 10.89.

10.91 Consider the oscillator in Fig. 10.133.

(a) Determine the oscillation frequency.

(b) Obtain the minimum value of R for which oscillation takes place.

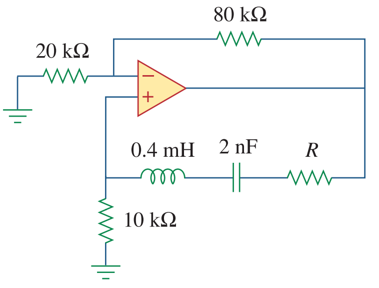


Figure 10.133
For Prob. 10.91.

10.92 The oscillator circuit in Fig. 10.134 uses an ideal op amp.

(a) Calculate the minimum value of R_o that will cause oscillation to occur.

(b) Find the frequency of oscillation.

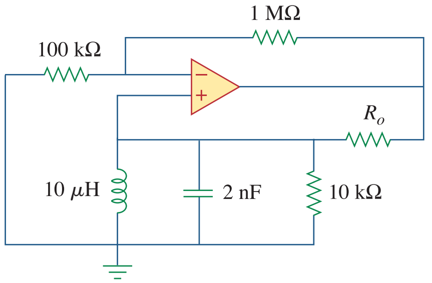


Figure 10.134
For Prob. 10.92.

10.93 Figure 10.135 shows a *Colpitts oscillator*. Show that the oscillation frequency is

$$f_o = \frac{1}{2\pi\sqrt{LC_T}}$$

where $C_T = C_1C_2/(C_1 + C_2)$. Assume $R_i \gg X_{C_2}$.

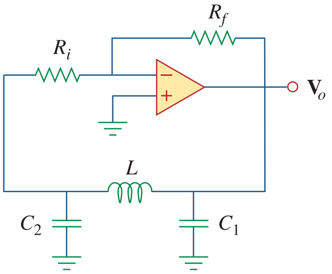


Figure 10.135
A Colpitts oscillator; for Prob. 10.93.

(*Hint:* Set the imaginary part of the impedance in the feedback circuit equal to zero.)

10.94 Design a Colpitts oscillator that will operate at 50 kHz.

10.95 Figure 10.136 shows a *Hartley oscillator*. Show that the frequency of oscillation is

$$f_o = \frac{1}{2\pi\sqrt{C(L_1 + L_2)}}$$

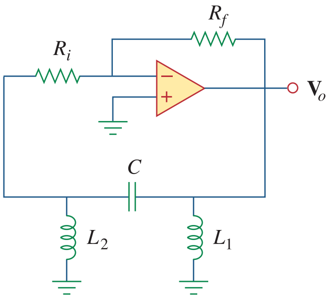


Figure 10.136
A Hartley oscillator; for Prob. 10.95.

10.96 Refer to the oscillator in Fig. 10.137.

(a) Show that

$$\frac{\mathbf{V}_2}{\mathbf{V}_o} = \frac{1}{3 + j(\omega L/R - R/\omega L)}$$

(b) Determine the oscillation frequency f_o.

(c) Obtain the relationship between R_1 and R_2 in order for oscillation to occur.

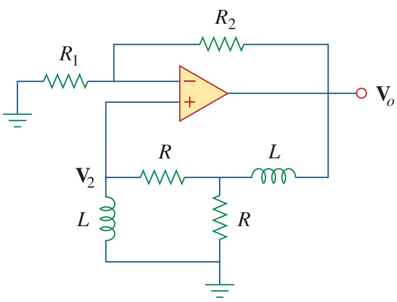


Figure 10.137
For Prob. 10.96.

Chapter 10, Problem 1.

Determine i in the circuit of Fig. 10.50.

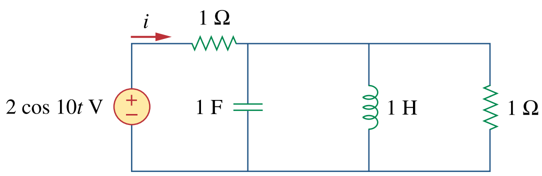


Figure 10.50
For Prob. 10.1.

Chapter 10, Solution 1.

We first determine the input impedance.

$$1H \longrightarrow j\omega L = j1x10 = j10$$

$$1F \longrightarrow 1/j\omega C = \frac{1}{j10x1} = -j0.1$$

$$Z_{in} = 1 + \left(\frac{1}{j10} + \frac{1}{-j0.1} + \frac{1}{1}\right)^{-1} = 1.0101 - j0.1 = 1.015 < -5.653^o$$

$$I = \frac{2 < 0^o}{1.015 < -5.653^o} = 1.9704 < 5.653^o$$

$$i(t) = \underline{1.9704\cos(10t + 5.653^o)\text{ A}} = \underline{\mathbf{1.9704cos(10t+5.65^\circ)\ A}}$$

Chapter 10, Problem 2.

Solve for $\mathbf{V}_o$ in Fig. 10.51, using nodal analysis.

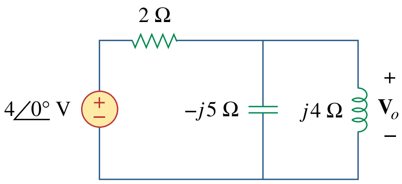
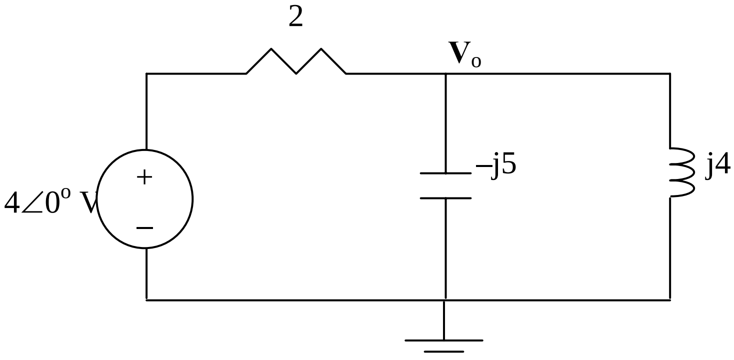


Figure 10.51
For Prob. 10.2.

Chapter 10, Solution 2.

Consider the circuit shown below.



At the main node,

$$\frac{4-V_o}{2} = \frac{V_o}{-j5} + \frac{V_o}{j4} \quad \longrightarrow \quad 40 = V_o(10 + j)$$

$$V_o = \frac{40}{10 - j} = \underline{3.98 < 5.71^o \quad A}$$

Chapter 10, Problem 3.

Determine v_o in the circuit of Fig. 10.52.

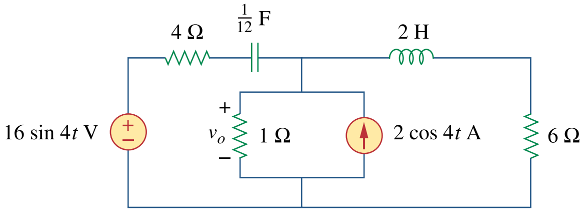


Figure 10.52
For Prob. 10.3.

Chapter 10, Solution 3.

$$\omega = 4$$

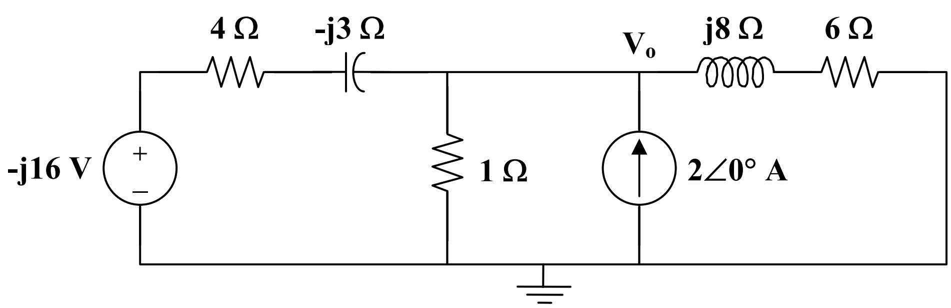
$$2\cos(4t) \longrightarrow 2\angle 0°$$

$$16\sin(4t) \longrightarrow 16\angle -90° = -j16$$

$$2\text{ H} \longrightarrow j\omega L = j8$$

$$1/12\text{ F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(4)(1/12)} = -j3$$

The circuit is shown below.



Applying nodal analysis,

$$\frac{-j16 - V_o}{4 - j3} + 2 = \frac{V_o}{1} + \frac{V_o}{6 + j8}$$

$$\frac{-j16}{4 - j3} + 2 = \left(1 + \frac{1}{4 - j3} + \frac{1}{6 + j8}\right) V_o$$

$$V_o = \frac{3.92 - j2.56}{1.22 + j0.04} = \frac{4.682\angle -33.15°}{1.2207\angle 1.88°} = 3.835\angle -35.02°$$

Therefore, $v_o(t) =$ **3.835 cos(4t – 35.02°) V**

Chapter 10, Problem 4.

Determine i_1 in the circuit of Fig. 10.53.

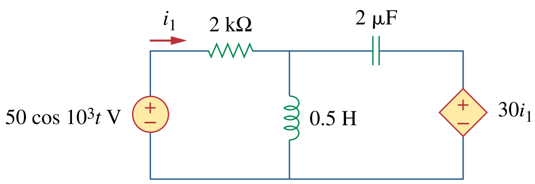


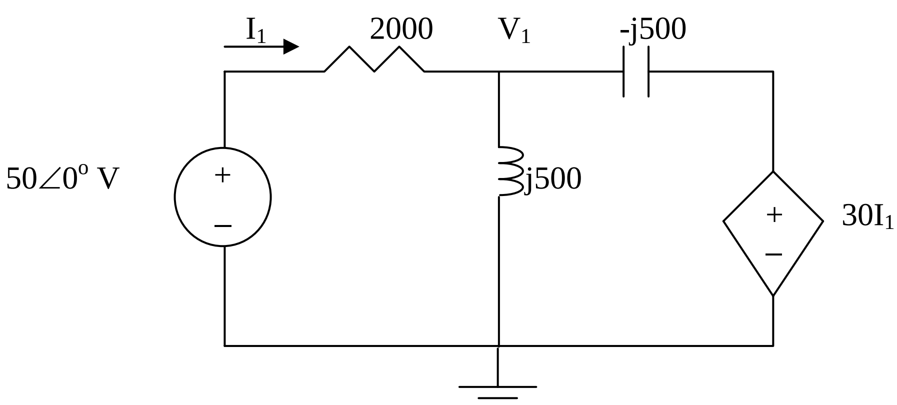
Figure 10.53
For Prob. 10.4.

Chapter 10, Solution 4.

$$0.5H \longrightarrow j\omega L = j0.5x10^3 = j500$$

$$2\mu F \longrightarrow \frac{1}{j\omega C} = \frac{1}{j10^3 x2x10^{-6}} = -j500$$

Consider the circuit as shown below.



At node 1,

$$\frac{50-V_1}{2000} + \frac{30I_1 - V_1}{-j500} = \frac{V_1}{j500}$$

But $I_1 = \dfrac{50-V_1}{2000}$

$$50 - V_1 + j4x30(\frac{50-V_1}{2000}) + j4V_1 - j4V_1 = 0 \quad \rightarrow \quad V_1 = 50$$

$$I_1 = \frac{50-V_1}{2000} = 0$$

$$i_1(t) = \underline{0\ A}$$

Chapter 10, Problem 5.

Find i_o in the circuit of Fig. 10.54.

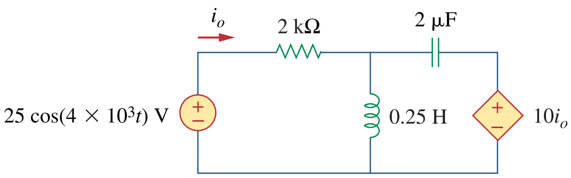
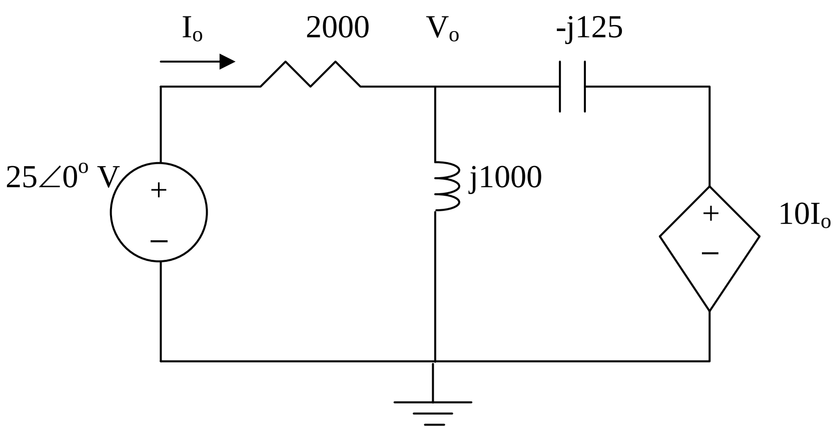


Figure 10.54
For Prob. 10.5.

Chapter 10, Solution 5.

$$0.25H \longrightarrow j\omega L = j0.25x4x10^3 = j1000$$
$$2\mu F \longrightarrow 1/j\omega C = \frac{1}{j4x10^3 x2x10^{-6}} = -j125$$

Consider the circuit as shown below.



At node V_o,

$$\frac{V_o - 25}{2000} + \frac{V_o - 0}{j1000} + \frac{V_o - 10I_o}{-j125} = 0$$
$$V_o - 25 - j2V_o + j16V_o - j160I_o = 0$$
$$(1 + j14)V_o - j160I_o = 25$$

But $I_o = (25–V_o)/2000$

$$(1 + j14)V_o - j2 + j0.08V_o = 25$$
$$V_o = \frac{25 + j2}{1 + j14.08} = \frac{25.08\angle 4.57°}{14.115\angle 58.94°} 1.7768\angle -81.37°$$

Now to solve for i_o,

$$I_o = \frac{25 - V_o}{2000} = \frac{25 - 0.2666 + j1.7567}{2000} = 12.367 + j0.8784\,\text{mA}$$
$$= 12.398\angle 4.06°$$

i_o = **<u>12.398cos(4x10^{3}t + 4.06°) mA</u>**.

Chapter 10, Problem 6.

Determine $\mathbf{V}_x$ in Fig. 10.55.

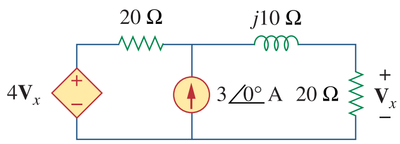


Figure 10.55
For Prob. 10.6.

Chapter 10, Solution 6.

Let V_o be the voltage across the current source. Using nodal analysis we get:

$$\frac{V_o - 4V_x}{20} - 3 + \frac{V_o}{20 + j10} = 0 \quad \text{where} \quad V_x = \frac{20}{20 + j10} V_o$$

Combining these we get:

$$\frac{V_o}{20} - \frac{4V_o}{20 + j10} - 3 + \frac{V_o}{20 + j10} = 0 \rightarrow (1 + j0.5 - 3)V_o = 60 + j30$$

$$V_o = \frac{60 + j30}{-2 + j0.5} \text{ or } V_x = \frac{20(3)}{-2 + j0.5} = \underline{\mathbf{29.11\angle{-166}^\circ\ V}}.$$

Chapter 10, Problem 7.

Use nodal analysis to find **V** in the circuit of Fig. 10.56.

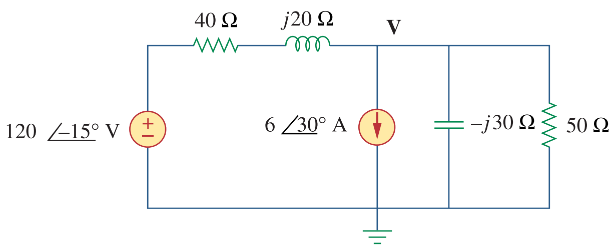


Figure 10.56
For Prob. 10.7.

Chapter 10, Solution 7.

At the main node,

$$\frac{120\angle -15^o - V}{40 + j20} = 6\angle 30^o + \frac{V}{-j30} + \frac{V}{50} \quad \longrightarrow \quad \frac{115.91 - j31.058}{40 + j20} - 5.196 - j3 =$$

$$V\left(\frac{1}{40 + j20} + \frac{j}{30} + \frac{1}{50}\right)$$

$$V = \frac{-3.1885 - j4.7805}{0.04 + j0.0233} = \underline{124.08\angle -154^o \text{ V}}$$

Chapter 10, Problem 8.

Use nodal analysis to find current i_o in the circuit of Fig. 10.57. Let $i_s = 6\cos(200t + 15°)$ A.

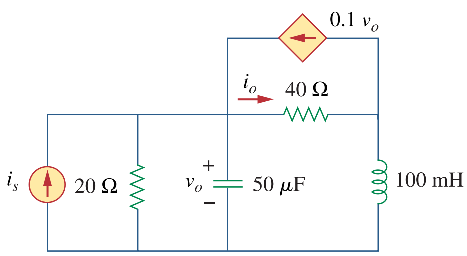


Figure 10.57
For Prob. 10.8.

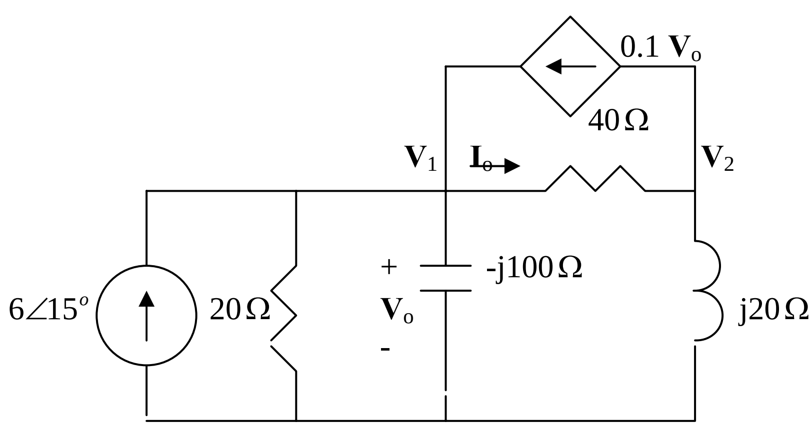
Chapter 10, Solution 8.

$\omega = 200,$

$100\text{mH} \longrightarrow \quad j\omega L = j200\text{x}0.1 = j20$

$50\mu\text{F} \longrightarrow \quad \dfrac{1}{j\omega C} = \dfrac{1}{j200\text{x}50\text{x}10^{-6}} = -j100$

The frequency-domain version of the circuit is shown below.



At node 1,

$$6\angle 15^{o} + 0.1V_1 = \frac{V_1}{20} + \frac{V_1}{-j100} + \frac{V_1 - V_2}{40}$$

or $$5.7955 + j1.5529 = (-0.025 + j0.01)V_1 - 0.025V_2 \qquad (1)$$

At node 2,

$$\frac{V_1 - V_2}{40} = 0.1V_1 + \frac{V_2}{j20} \longrightarrow 0 = 3V_1 + (1 - j2)V_2 \qquad (2)$$

From (1) and (2),

$$\begin{bmatrix} (-0.025 + j0.01) & -0.025 \\ 3 & (1 - j2) \end{bmatrix} \begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \begin{pmatrix} (5.7955 + j1.5529) \\ 0 \end{pmatrix} \quad \text{or} \quad AV = B$$

Using MATLAB,

V = inv(A)*B

leads to $V_1 = -70.63 - j127.23, \qquad V_2 = -110.3 + j161.09$

$$I_o = \frac{V_1 - V_2}{40} = 7.276\angle -82.17^{o}$$

Thus,

$$\underline{i_o(t) = 7.276\cos(200t - 82.17^{o})\ \text{A}}$$

Chapter 10, Problem 9.

Use nodal analysis to find v_o in the circuit of Fig. 10.58.

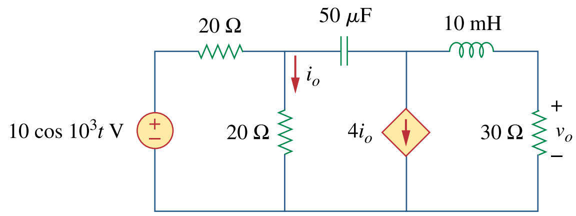
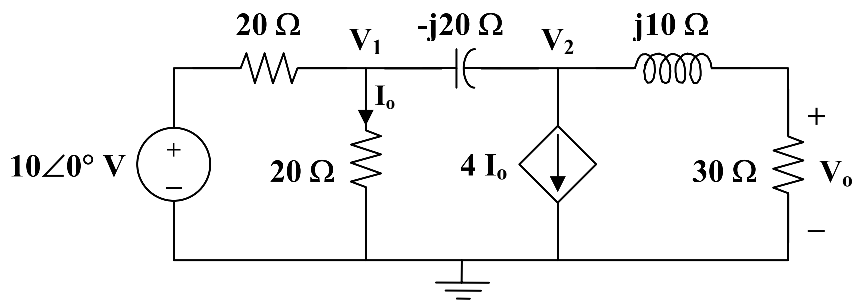


Figure 10.58
For Prob. 10.9.

Chapter 10, Solution 9.

$$10\cos(10^3 t) \longrightarrow 10\angle 0°, \quad \omega = 10^3$$
$$10 \text{ mH} \longrightarrow j\omega L = j10$$
$$50\ \mu\text{F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(10^3)(50\times 10^{-6})} = -j20$$

Consider the circuit shown below.



At node 1,

$$\frac{10 - \mathbf{V}_1}{20} = \frac{\mathbf{V}_1}{20} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{-j20}$$
$$10 = (2 + j)\mathbf{V}_1 - j\mathbf{V}_2 \qquad (1)$$

At node 2,

$$\frac{\mathbf{V}_1 - \mathbf{V}_2}{-j20} = (4)\frac{\mathbf{V}_1}{20} + \frac{\mathbf{V}_2}{30 + j10}, \text{ where } \mathbf{I}_o = \frac{\mathbf{V}_1}{20} \text{ has been substituted.}$$
$$(-4 + j)\mathbf{V}_1 = (0.6 + j0.8)\mathbf{V}_2$$
$$\mathbf{V}_1 = \frac{0.6 + j0.8}{-4 + j}\mathbf{V}_2 \qquad (2)$$

Substituting (2) into (1)

$$10 = \frac{(2 + j)(0.6 + j0.8)}{-4 + j}\mathbf{V}_2 - j\mathbf{V}_2$$

or

$$\mathbf{V}_2 = \frac{170}{0.6 - j26.2}$$

$$\mathbf{V}_o = \frac{30}{30 + j10}\mathbf{V}_2 = \frac{3}{3 + j} \cdot \frac{170}{0.6 - j26.2} = 6.154\angle 70.26°$$

Therefore, $v_o(t) =$ <u>**$6.154\cos(10^3 t + 70.26°)$ V**</u>

Chapter 10, Problem 10.

Use nodal analysis to find v_o in the circuit of Fig. 10.59. Let $\omega = 2$ krad/s.

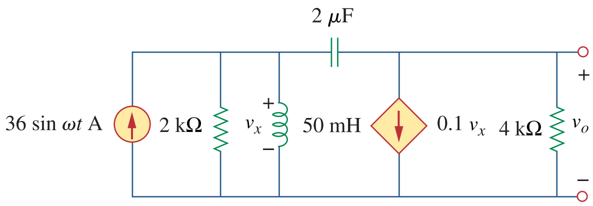


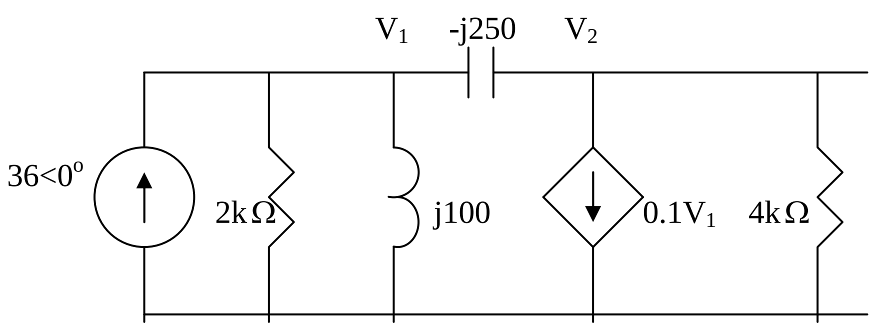
Figure 10.59
For Prob. 10.10.

Chapter 10, Solution 10.

$$50\text{ mH} \longrightarrow j\omega L = j2000x50x10^{-3} = j100, \qquad \omega = 2000$$

$$2\mu F \longrightarrow \frac{1}{j\omega C} = \frac{1}{j2000x2x10^{-6}} = -j250$$

Consider the frequency-domain equivalent circuit below.



At node 1,

$$36 = \frac{V_1}{2000} + \frac{V_1}{j100} + \frac{V_1 - V_2}{-j250} \longrightarrow 36 = (0.0005 - j0.006)V_1 - j0.004V_2 \qquad (1)$$

At node 2,

$$\frac{V_1 - V_2}{-j250} = 0.1V_1 + \frac{V_2}{4000} \longrightarrow 0 = (0.1 - j0.004)V_1 + (0.00025 + j0.004)V_2 \quad (2)$$

Solving (1) and (2) gives

$$V_o = V_2 = -535.6 + j893.5 = 8951.1\angle 93.43^o$$

v_o (t) = **8.951 sin(2000t +93.43°) kV**

Chapter 10, Problem 11.

Apply nodal analysis to the circuit in Fig. 10.60 and determine $\mathbf{I}_o$.

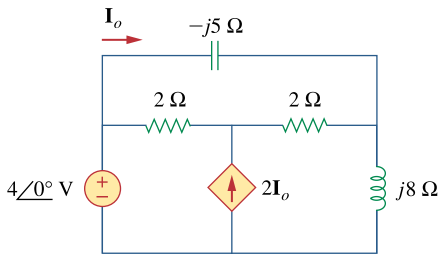
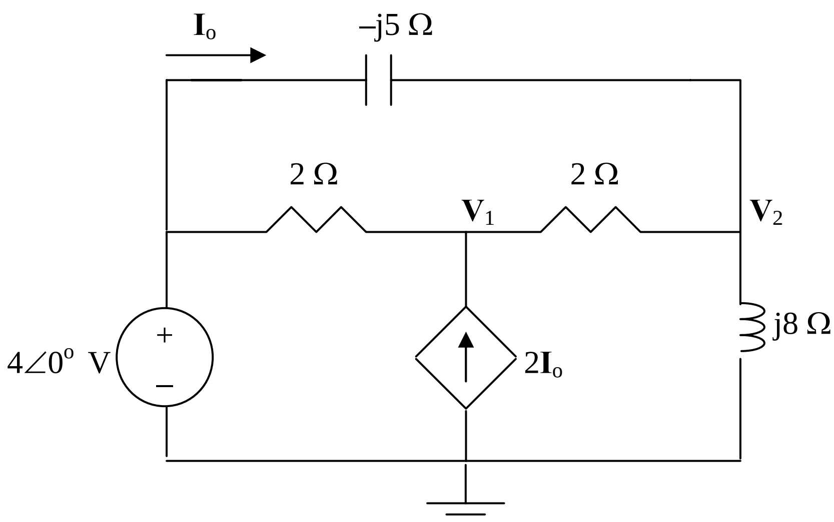


Figure 10.60
For Prob. 10.11.

Chapter 10, Solution 11.

Consider the circuit as shown below.



At node 1,

$$\frac{V_1 - 4}{2} - 2I_o + \frac{V_1 - V_2}{2} = 0$$

$$V_1 - 0.5V_2 - 2I_o = 2$$

But, $I_o = (4–V_2)/(–j5) = –j0.2V_2 + j0.8$

Now the first node equation becomes,

$V_1 – 0.5V_2 + j0.4V_2 – j1.6 = 2$ or
$V_1 + (–0.5+j0.4)V_2 = 2 + j1.6$

At node 2,

$$\frac{V_2 - V_1}{2} + \frac{V_2 - 4}{-j5} + \frac{V_2 - 0}{j8} = 0$$

$$–0.5V_1 + (0.5 + j0.075)V_2 = j0.8$$

Using MATLAB to solve this, we get,

```
>> Y=[1,(-0.5+0.4i);-0.5,(0.5+0.075i)]

Y =

  1.0000           -0.5000 + 0.4000i
 -0.5000            0.5000 + 0.0750i

>> I=[(2+1.6i);0.8i]

I =

  2.0000 + 1.6000i
       0 + 0.8000i

>> V=inv(Y)*I

V =

  4.8597 + 0.0543i
  4.9955 + 0.9050i
```

$I_o = –j0.2V_2 + j0.8 = –j0.9992 + 0.01086 + j0.8 = 0.01086 – j0.1992$

$= \underline{\mathbf{199.5\angle 86.89^\circ\ mA}}$.

Chapter 10, Problem 12.

By nodal analysis, find i_o in the circuit of Fig. 10.61.

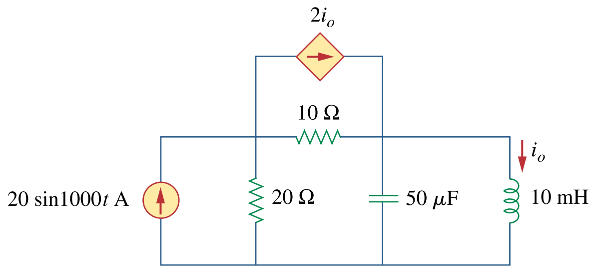


Figure 10.61
For Prob. 10.12.

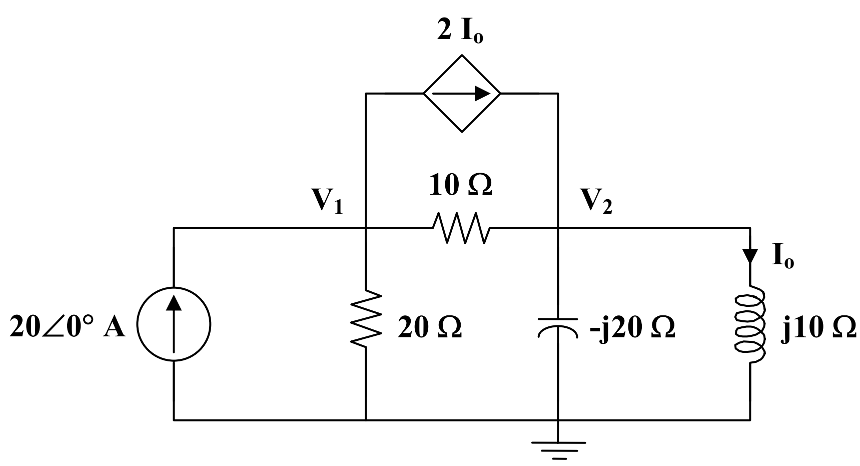
Chapter 10, Solution 12.

$$20\sin(1000t) \longrightarrow 20\angle 0°, \quad \omega = 1000$$

$$10 \text{ mH} \longrightarrow j\omega L = j10$$

$$50\ \mu\text{F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(10^3)(50\times 10^{-6})} = -j20$$

The frequency-domain equivalent circuit is shown below.



At node 1,

$$20 = 2\mathbf{I}_o + \frac{\mathbf{V}_1}{20} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{10},$$ where

$$\mathbf{I}_o = \frac{\mathbf{V}_2}{j10}$$

$$20 = \frac{2\mathbf{V}_2}{j10} + \frac{\mathbf{V}_1}{20} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{10}$$

$$400 = 3\mathbf{V}_1 - (2 + j4)\,\mathbf{V}_2 \qquad (1)$$

At node 2,

$$\frac{2\mathbf{V}_2}{j10} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{10} = \frac{\mathbf{V}_2}{-j20} + \frac{\mathbf{V}_2}{j10}$$

$$j2\,\mathbf{V}_1 = (-3 + j2)\,\mathbf{V}_2$$

or

$$\mathbf{V}_1 = (1 + j1.5)\,\mathbf{V}_2 \qquad (2)$$

Substituting (2) into (1),

$$400 = (3 + j4.5)\,\mathbf{V}_2 - (2 + j4)\,\mathbf{V}_2 = (1 + j0.5)\,\mathbf{V}_2$$

$$\mathbf{V}_2 = \frac{400}{1 + j0.5}$$

$$\mathbf{I}_o = \frac{\mathbf{V}_2}{j10} = \frac{40}{j(1 + j0.5)} = 35.74\angle -116.6^\circ$$

Therefore, $i_o(t) =$ **35.74 sin(1000t – 116.6°) A**

Chapter 10, Problem 14.

Calculate the voltage at nodes 1 and 2 in the circuit of Fig. 10.63 using nodal analysis.

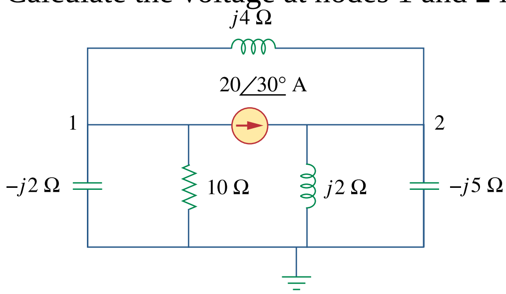


Figure 10.63
For Prob. 10.14.

Chapter 10, Solution 14.
At node 1,

$$\frac{0-\mathbf{V}_1}{-j2}+\frac{0-\mathbf{V}_1}{10}+\frac{\mathbf{V}_2-\mathbf{V}_1}{j4}=20\angle 30°$$

$$-(1+j2.5)\,\mathbf{V}_1 - j2.5\,\mathbf{V}_2 = 173.2 + j100 \qquad (1)$$

At node 2,

$$\frac{\mathbf{V}_2}{j2}+\frac{\mathbf{V}_2}{-j5}+\frac{\mathbf{V}_2-\mathbf{V}_1}{j4}=20\angle 30°$$

$$-j5.5\,\mathbf{V}_2 + j2.5\,\mathbf{V}_1 = 173.2 + j100 \qquad (2)$$

Equations (1) and (2) can be cast into matrix form as

$$\begin{bmatrix} 1+j2.5 & j2.5 \\ j2.5 & -j5.5 \end{bmatrix}\begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix} = \begin{bmatrix} -200\angle 30° \\ 200\angle 30° \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 1+j2.5 & j2.5 \\ j2.5 & -j5.5 \end{vmatrix} = 20 - j5.5 = 20.74\angle -15.38°$$

$$\Delta_1 = \begin{vmatrix} -200\angle 30° & j2.5 \\ 200\angle 30° & -j5.5 \end{vmatrix} = j3(200\angle 30°) = 600\angle 120°$$

$$\Delta_2 = \begin{vmatrix} 1+j2.5 & -200\angle 30° \\ j2.5 & 200\angle 30° \end{vmatrix} = (200\angle 30°)(1+j5) = 1020\angle 108.7°$$

$$\mathbf{V}_1 = \frac{\Delta_1}{\Delta} = 28.93\angle 135.38°$$

$$\mathbf{V}_2 = \frac{\Delta_2}{\Delta} = 49.18\angle 124.08°$$

Chapter 10, Problem 15.

Solve for the current **I** in the circuit of Fig. 10.64 using nodal analysis.

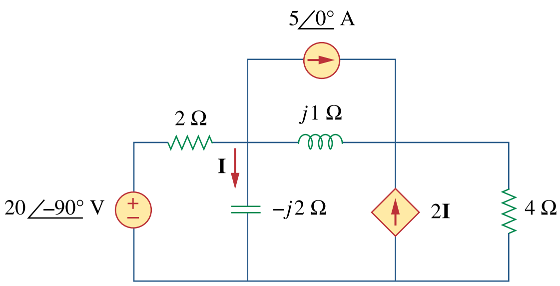
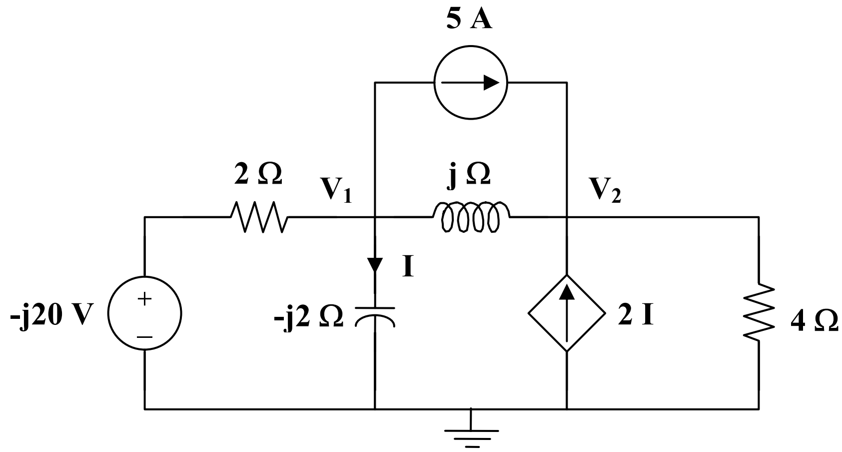


Figure 10.64
For Prob. 10.15.

Chapter 10, Solution 15.

We apply nodal analysis to the circuit shown below.



At node 1,

$$\frac{-j20-\mathbf{V}_1}{2}=5+\frac{\mathbf{V}_1}{-j2}+\frac{\mathbf{V}_1-\mathbf{V}_2}{j}$$

$$-5-j10=(0.5-j0.5)\,\mathbf{V}_1+j\,\mathbf{V}_2 \qquad (1)$$

At node 2,

$$5+2\mathbf{I}+\frac{\mathbf{V}_1-\mathbf{V}_2}{j}=\frac{\mathbf{V}_2}{4},$$

where $\mathbf{I}=\dfrac{\mathbf{V}_1}{-j2}$

$$\mathbf{V}_2=\frac{5}{0.25-j}\,\mathbf{V}_1 \qquad (2)$$

Substituting (2) into (1),

$$-5-j10-\frac{j5}{0.25-j}=0.5(1-j)\,\mathbf{V}_1$$

$$(1-j)\,\mathbf{V}_1=-10-j20-\frac{j40}{1-j4}$$

$$(\sqrt{2}\angle -45°)\,\mathbf{V}_1=-10-j20+\frac{160}{17}-\frac{j40}{17}$$

$$\mathbf{V}_1=15.81\angle 313.5°$$

$$\mathbf{I}=\frac{\mathbf{V}_1}{-j2}=(0.5\angle 90°)(15.81\angle 313.5°)$$

$$\mathbf{I}=\underline{7.906\angle 43.49°\text{ A}}$$

Chapter 10, Problem 16.

Use nodal analysis to find $\mathbf{V}_x$ in the circuit shown in Fig. 10.65.

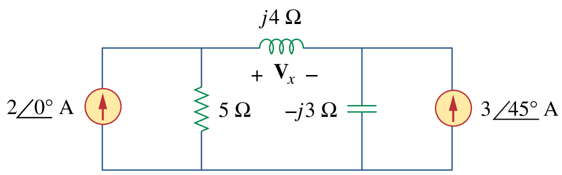
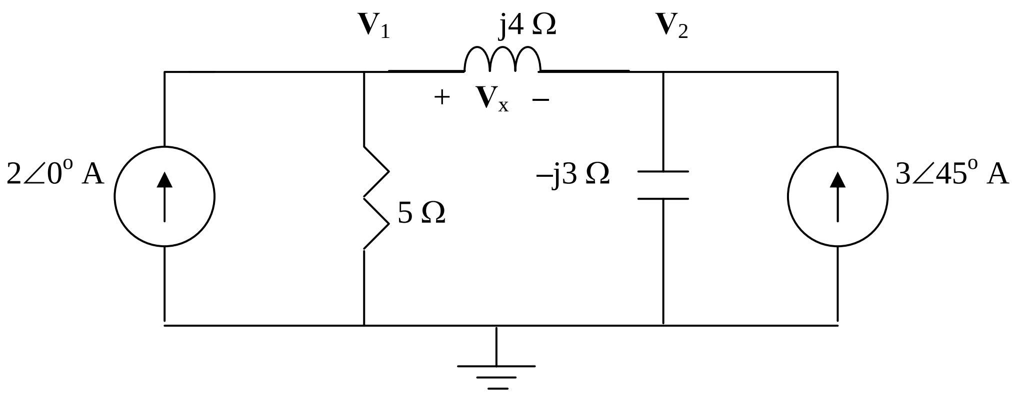


Figure 10.65
For Prob. 10.16.

Chapter 10, Solution 16.

Consider the circuit as shown in the figure below.



At node 1,

$$-2+\frac{V_1-0}{5}+\frac{V_1-V_2}{j4}=0 \qquad (1)$$

$$(0.2-j0.25)V_1+j0.25V_2=2$$

At node 2,

$$\frac{V_2-V_1}{j4}+\frac{V_2-0}{-j3}-3\angle 45^\circ=0 \qquad (2)$$

$$j0.25V_1+j0.08333V_2=2.121+j2.121$$

In matrix form, (1) and (2) become

$$\begin{bmatrix} 0.2-j0.25 & j0.25 \\ j0.25 & j0.08333 \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix}=\begin{bmatrix} 2 \\ 2.121+j2.121 \end{bmatrix}$$

Solving this using MATLAB, we get,

```
>> Y=[(0.2-0.25i),0.25i;0.25i,0.08333i]

Y =

   0.2000 - 0.2500i        0 + 0.2500i
        0 + 0.2500i        0 + 0.0833i

>> I=[2;(2.121+2.121i)]

I =

   2.0000
   2.1210 + 2.1210i

>> V=inv(Y)*I

V =

   5.2793 - 5.4190i
   9.6145 - 9.1955i
```

$V_s = V_1 - V_2 = -4.335 + j3.776 =$ **$\underline{5.749\angle 138.94^\circ \text{ V}}$**.

Chapter 10, Problem 17.

By nodal analysis, obtain current $\mathbf{I}_o$ in the circuit of Fig. 10.66.

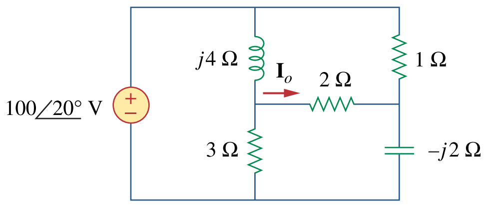
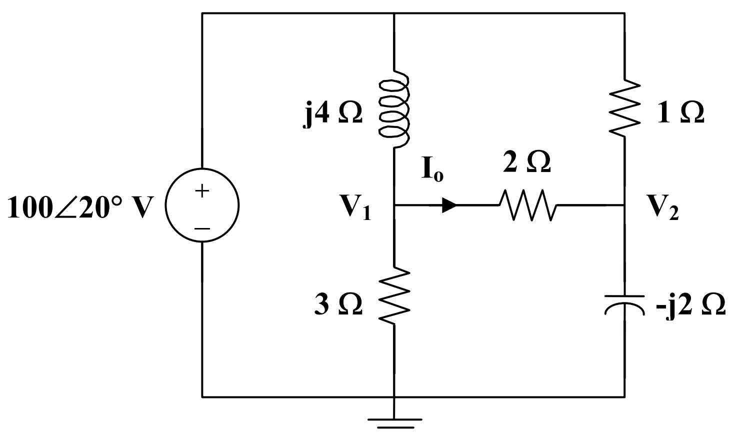


Figure 10.66
For Prob. 10.17.

Chapter 10, Solution 17.

Consider the circuit below.



At node 1,

$$\frac{100\angle 20° - \mathbf{V}_1}{j4} = \frac{\mathbf{V}_1}{3} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{2}$$

$$100\angle 20° = \frac{\mathbf{V}_1}{3}(3 + j10) - j2\,\mathbf{V}_2$$

(1)

At node 2,

$$\frac{100\angle 20° - \mathbf{V}_2}{1} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{2} = \frac{\mathbf{V}_2}{-j2}$$

$$100\angle 20° = -0.5\,\mathbf{V}_1 + (1.5 + j0.5)\,\mathbf{V}_2$$

(2)

From (1) and (2),

$$\begin{bmatrix} 100\angle 20° \\ 100\angle 20° \end{bmatrix} = \begin{bmatrix} -0.5 & 0.5(3+j) \\ 1 + j10/3 & -j2 \end{bmatrix} \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} -0.5 & 1.5 + j0.5 \\ 1 + j10/3 & -j2 \end{vmatrix} = 0.1667 - j4.5$$

$$\Delta_1 = \begin{vmatrix} 100\angle 20° & 1.5 + j0.5 \\ 100\angle 20° & -j2 \end{vmatrix} = -55.45 - j286.2$$

$$\Delta_2 = \begin{vmatrix} -0.5 & 100\angle 20° \\ 1 + j10/3 & 100\angle 20° \end{vmatrix} = -26.95 - j364.5$$

$$\mathbf{V}_1 = \frac{\Delta_1}{\Delta} = 64.74\angle -13.08°$$

$$\mathbf{V}_2 = \frac{\Delta_2}{\Delta} = 81.17\angle -6.35°$$

$$\mathbf{I}_o = \frac{\mathbf{V}_1 - \mathbf{V}_2}{2} = \frac{\Delta_1 - \Delta_2}{2\Delta} = \frac{-28.5 + j78.31}{0.3333 - j9}$$

$$\mathbf{I}_o = \underline{\mathbf{9.25\angle -162.12°\ A}}$$

Chapter 10, Problem 18.

Use nodal analysis to obtain $\mathbf{V}_o$ in the circuit of Fig. 10.67 below.

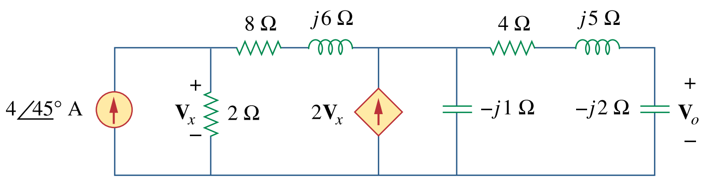
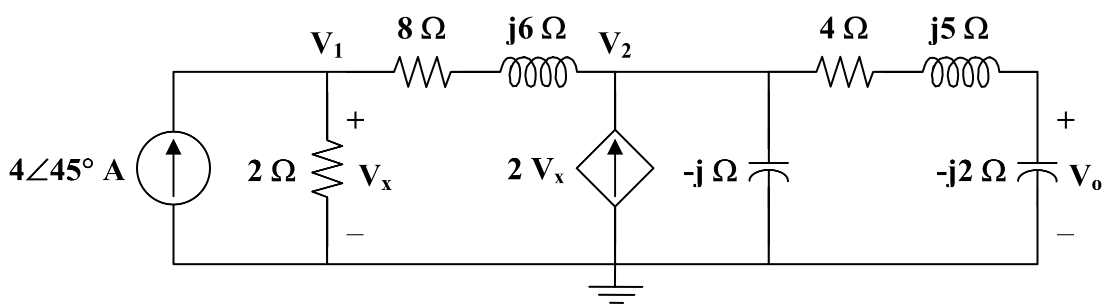


Figure 10.67
For Prob. 10.18.

Chapter 10, Solution 18.

Consider the circuit shown below.



At node 1,

$$4\angle 45° = \frac{\mathbf{V}_1}{2} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{8 + j6}$$

$$200\angle 45° = (29 - j3)\,\mathbf{V}_1 - (4 - j3)\,\mathbf{V}_2 \qquad (1)$$

At node 2,

$$\frac{\mathbf{V}_1 - \mathbf{V}_2}{8 + j6} + 2\mathbf{V}_x = \frac{\mathbf{V}_2}{-j} + \frac{\mathbf{V}_2}{4 + j5 - j2}, \qquad \text{where} \quad \mathbf{V}_x = \mathbf{V}_1$$

$$(104 - j3)\,\mathbf{V}_1 = (12 + j41)\,\mathbf{V}_2$$

$$\mathbf{V}_1 = \frac{12 + j41}{104 - j3}\mathbf{V}_2 \qquad (2)$$

Substituting (2) into (1),

$$200\angle 45° = (29 - j3)\frac{(12 + j41)}{104 - j3}\mathbf{V}_2 - (4 - j3)\,\mathbf{V}_2$$

$$200\angle 45° = (14.21\angle 89.17°)\,\mathbf{V}_2$$

$$\mathbf{V}_2 = \frac{200\angle 45°}{14.21\angle 89.17°}$$

$$\mathbf{V}_o = \frac{-j2}{4 + j5 - j2}\mathbf{V}_2 = \frac{-j2}{4 + j3}\mathbf{V}_2 = \frac{-6 - j8}{25}\mathbf{V}_2$$

$$\mathbf{V}_o = \frac{10\angle 233.13°}{25} \cdot \frac{200\angle 45°}{14.21\angle 89.17°}$$

$$\mathbf{V}_o = \underline{\mathbf{5.63\angle 189°\ V}}$$

Chapter 10, Problem 19.

Obtain $\mathbf{V}_o$ in Fig. 10.68 using nodal analysis.

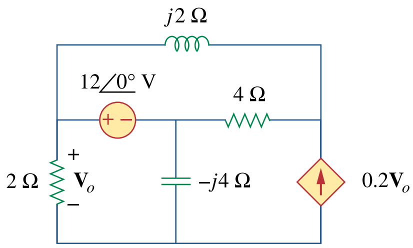


Figure 10.68
For Prob. 10.19.

Chapter 10, Problem 20.

Refer to Fig. 10.69. If $v_s(t) = V_m \sin \omega t$ and $v_o(t) = A\sin(\omega t + \phi)$ derive the expressions for A and ϕ

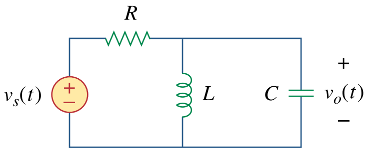
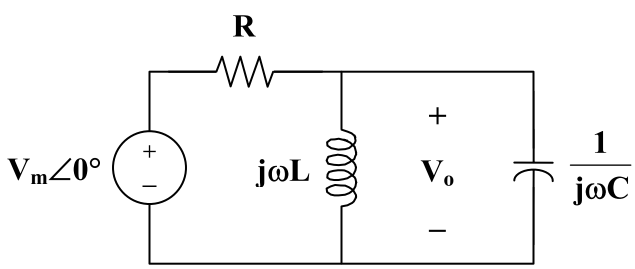


Figure 10.69
For Prob. 10.20.

Chapter 10, Solution 20.

The circuit is converted to its frequency-domain equivalent circuit as shown below.



Let $\quad \mathbf{Z} = j\omega L \,\|\, \frac{1}{j\omega C} = \frac{\frac{L}{C}}{j\omega L + \frac{1}{j\omega C}} = \frac{j\omega L}{1-\omega^2 LC}$

$$\mathbf{V}_o = \frac{\mathbf{Z}}{R+\mathbf{Z}} V_m = \frac{\frac{j\omega L}{1-\omega^2 LC}}{R + \frac{j\omega L}{1-\omega^2 LC}} V_m = \frac{j\omega L}{R(1-\omega^2 LC) + j\omega L} V_m$$

$$\mathbf{V}_o = \frac{\omega L\, V_m}{\sqrt{R^2(1-\omega^2 LC)^2 + \omega^2 L^2}} \angle\left(90° - \tan^{-1}\frac{\omega L}{R(1-\omega^2 LC)}\right)$$

If $\quad \mathbf{V}_o = A\angle\phi$, then

$$A = \underline{\boldsymbol{\frac{\omega L\, V_m}{\sqrt{R^2(1-\omega^2 LC)^2 + \omega^2 L^2}}}}$$

and $\quad \phi = \underline{\mathbf{90° - \tan^{-1}\frac{\omega L}{R(1-\omega^2 LC)}}}$

Chapter 10, Problem 21.

For each of the circuits in Fig. 10.70, find $\mathbf{V}_o/\mathbf{V}_i$ for $\omega = 0, \omega \to \infty$, and $\omega^2 = 1/LC$.

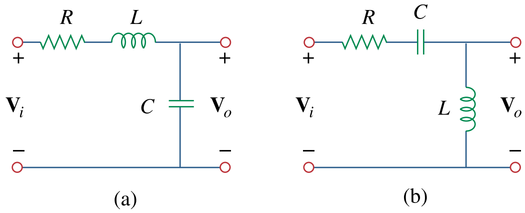


Figure 10.70
For Prob. 10.21.

Chapter 10, Solution 21.

(a) $$\frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{\dfrac{1}{j\omega C}}{R + j\omega L + \dfrac{1}{j\omega C}} = \frac{1}{1 - \omega^2 LC + j\omega RC}$$

At $\omega = 0$, $$\frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{1}{1} = \underline{\mathbf{1}}$$

As $\omega \to \infty$, $$\frac{\mathbf{V}_o}{\mathbf{V}_i} = \underline{\mathbf{0}}$$

At $\omega = \dfrac{1}{\sqrt{LC}}$, $$\frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{1}{jRC \cdot \dfrac{1}{\sqrt{LC}}} = \underline{\frac{\mathbf{-j}}{\mathbf{R}}\sqrt{\frac{\mathbf{L}}{\mathbf{C}}}}$$

(b) $$\frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{j\omega L}{R + j\omega L + \dfrac{1}{j\omega C}} = \frac{-\omega^2 LC}{1 - \omega^2 LC + j\omega RC}$$

At $\omega = 0$, $$\frac{\mathbf{V}_o}{\mathbf{V}_i} = \underline{\mathbf{0}}$$

As $\omega \to \infty$, $$\frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{1}{1} = \underline{\mathbf{1}}$$

At $\omega = \dfrac{1}{\sqrt{LC}}$, $$\frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{-1}{jRC \cdot \dfrac{1}{\sqrt{LC}}} = \underline{\frac{\mathbf{j}}{\mathbf{R}}\sqrt{\frac{\mathbf{L}}{\mathbf{C}}}}$$

Chapter 10, Problem 22.

For the circuit in Fig. 10.71, determine $\mathbf{V}_o/\mathbf{V}_s$.

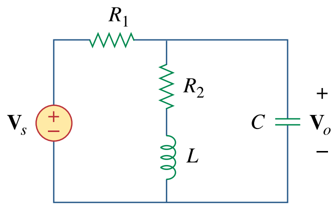
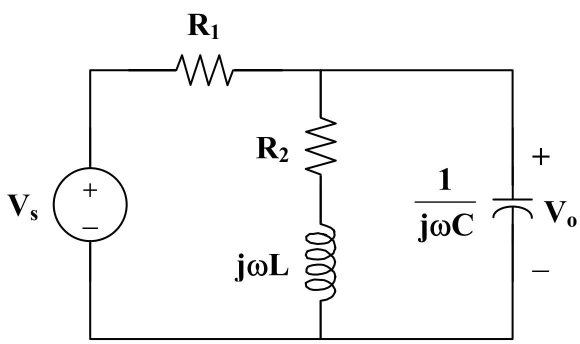


Figure 10.71
For Prob. 10.22.

Chapter 10, Solution 22.

Consider the circuit in the frequency domain as shown below.



Let $\quad \mathbf{Z} = (R_2 + j\omega L) \parallel \dfrac{1}{j\omega C}$

$$\mathbf{Z} = \frac{\dfrac{1}{j\omega C}(R_2 + j\omega L)}{R_2 + j\omega L + \dfrac{1}{j\omega C}} = \frac{R_2 + j\omega L}{1 + j\omega R_2 - \omega^2 LC}$$

$$\frac{\mathbf{V}_o}{\mathbf{V}_s} = \frac{\mathbf{Z}}{\mathbf{Z} + R_1} = \frac{\dfrac{R_2 + j\omega L}{1 - \omega^2 LC + j\omega R_2 C}}{R_1 + \dfrac{R_2 + j\omega L}{1 - \omega^2 LC + j\omega R_2 C}}$$

$$\frac{\mathbf{V}_o}{\mathbf{V}_s} = \underline{\frac{\mathbf{R_2 + j\omega L}}{\mathbf{R_1 + R_2 - \omega^2 LCR_1 + j\omega(L + R_1R_2C)}}}$$

Chapter 10, Problem 23.

Using nodal analysis obtain **V** in the circuit of Fig. 10.72.

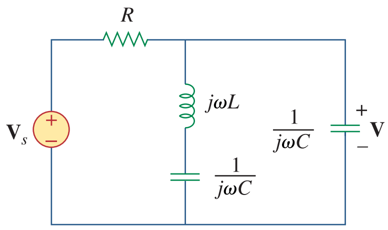


Figure 10.72
For Prob. 10.23.

Chapter 10, Solution 23.

$$\frac{V - V_s}{R} + \frac{V}{j\omega L + \frac{1}{j\omega C}} + j\omega CV = 0$$

$$V + \frac{j\omega RCV}{-\omega^2 LC + 1} + j\omega RCV = V_s$$

$$\left(\frac{1 - \omega^2 LC + j\omega RC + j\omega RC - j\omega^3 RLC^2}{1 - \omega^2 LC}\right) V = V_s$$

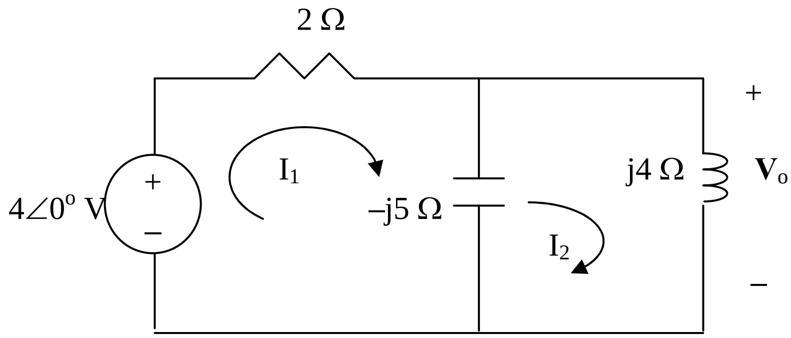
$$V = \underline{\frac{(1 - \omega^2 LC)V_s}{1 - \omega^2 LC + j\omega RC(2 - \omega^2 LC)}}$$

Chapter 10, Problem 24.

Use mesh analysis to find $\mathbf{V}_o$ in the circuit of Prob. 10.2.

Chapter 10, Solution 24.

Consider the circuit as shown below.



For mesh 1,

$$4 = (2 - j5)I_1 + j5I_1 \qquad (1)$$

For mesh 2,

$$0 = j5I_1 + (j4 - j5)I_2 \quad \longrightarrow \quad I_1 = \frac{1}{5}I_2 \qquad (2)$$

Substituting (2) into (1),

$$4 = (2 - j5)\frac{1}{5}I_2 + j5I_2 \quad \longrightarrow \quad I_2 = \frac{1}{0.1 + j}$$

$$V_o = j4I_2 = \frac{j4}{0.1 + j} = \underline{3.98 < 5.71^o \text{ V}}$$

Chapter 10, Problem 25.

ML

Solve for i_o in Fig. 10.73 using mesh analysis.

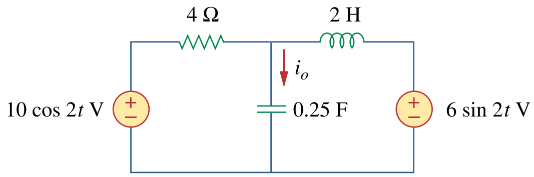
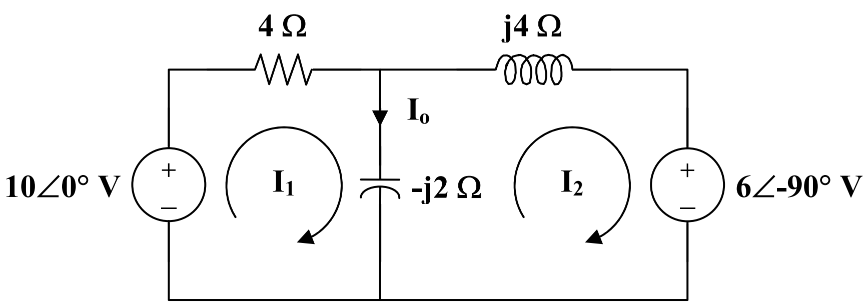


Figure 10.73
For Prob. 10.25.

Chapter 10, Solution 25.

$$\omega = 2$$
$$10\cos(2t) \longrightarrow 10\angle 0°$$
$$6\sin(2t) \longrightarrow 6\angle -90° = -j6$$
$$2\text{ H} \longrightarrow j\omega L = j4$$
$$0.25\text{ F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(2)(1/4)} = -j2$$

The circuit is shown below.



For loop 1,

$$-10 + (4 - j2)\mathbf{I}_1 + j2\mathbf{I}_2 = 0$$
$$5 = (2 - j)\mathbf{I}_1 + j\mathbf{I}_2 \qquad (1)$$

For loop 2,

$$j2\mathbf{I}_1 + (j4 - j2)\mathbf{I}_2 + (-j6) = 0$$
$$\mathbf{I}_1 + \mathbf{I}_2 = 3 \qquad (2)$$

In matrix form (1) and (2) become

$$\begin{bmatrix} 2-j & j \\ 1 & 1 \end{bmatrix}\begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} 5 \\ 3 \end{bmatrix}$$

$$\Delta = 2(1-j), \qquad \Delta_1 = 5 - j3, \qquad \Delta_2 = 1 - j3$$

$$\mathbf{I}_o = \mathbf{I}_1 - \mathbf{I}_2 = \frac{\Delta_1 - \Delta_2}{\Delta} = \frac{4}{2(1-j)} = 1 + j = 1.414\angle 45°$$

Therefore, $i_o(t) =$ **$\underline{1.4142\cos(2t + 45°)\text{ A}}$**

Chapter 10, Problem 26.

Use mesh analysis to find current i_o in the circuit of Fig. 10.74.

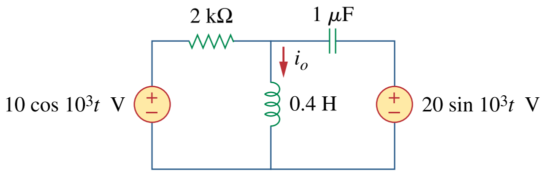


Figure 10.74
For Prob. 10.26.

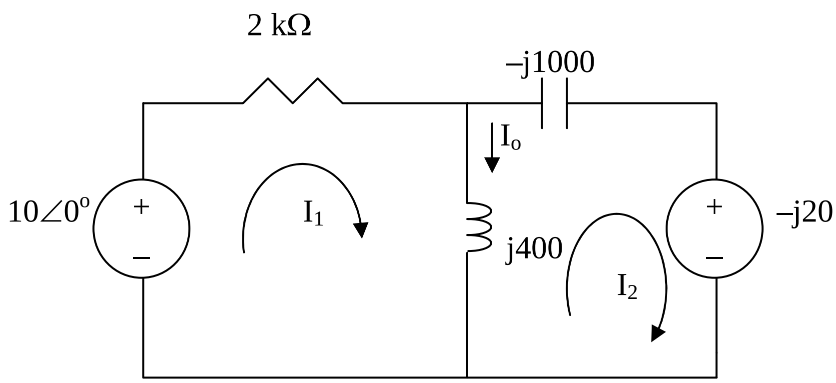
Chapter 10, Solution 26.

$$0.4H \longrightarrow j\omega L = j10^3 x0.4 = j400$$

$$1\mu F \longrightarrow \frac{1}{j\omega C} = \frac{1}{j10^3 x10^{-6}} = -j1000$$

$$20\sin 10^3 t = 20\cos(10^3 t - 90^o) \longrightarrow 20 < -90 = -j20$$

The circuit becomes that shown below.



For loop 1,

$$-10 + (12000 + j400)I_1 - j400I_2 = 0 \longrightarrow 1 = (200 + j40)I_1 - j40I_2 \ (1)$$

For loop 2,

$$-j20 + (j400 - j1000)I_2 - j400I_1 = 0 \longrightarrow -12 = 40I_1 + 60I_2$$

(2)

In matrix form, (1) and (2) become

$$\begin{bmatrix} 1 \\ -12 \end{bmatrix} = \begin{bmatrix} 200 + j40 & -j40 \\ 40 & 60 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

Solving this leads to

I_1 =0.0025-j0.0075, I_2 = -0.035+j0.005

$$I_o = I_1 - I_2 = 0.0375 - j0.0125 = 39.5 < -18.43 \text{ mA}$$

$$i_o = \underline{39.5\cos(10^3 t - 18.43^o) \text{ mA}}$$

Chapter 10, Problem 27.

Using mesh analysis, find $\mathbf{I}_1$ and $\mathbf{I}_2$ in the circuit of Fig. 10.75.

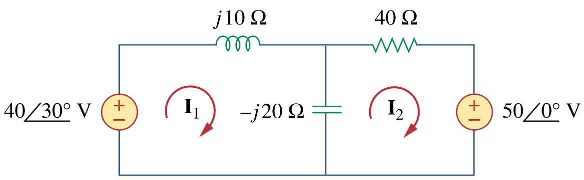


Figure 10.75
For Prob. 10.27.

Chapter 10, Solution 27.

For mesh 1,

$$-40\angle 30^\circ + (j10 - j20)\mathbf{I}_1 + j20\mathbf{I}_2 = 0$$
$$4\angle 30^\circ = -j\mathbf{I}_1 + j2\mathbf{I}_2 \qquad (1)$$

For mesh 2,

$$50\angle 0^\circ + (40 - j20)\mathbf{I}_2 + j20\mathbf{I}_1 = 0$$
$$5 = -j2\mathbf{I}_1 - (4 - j2)\mathbf{I}_2 \qquad (2)$$

From (1) and (2),

$$\begin{bmatrix} 4\angle 30^\circ \\ 5 \end{bmatrix} = \begin{bmatrix} -j & j2 \\ -j2 & -(4-j2) \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix}$$

$$\Delta = -2 + 4j = 4.472\angle 116.56^\circ$$

$$\Delta_1 = -(4\angle 30^\circ)(4 - j2) - j10 = 21.01\angle 211.8^\circ$$

$$\Delta_2 = -j5 + 8\angle 120^\circ = 4.44\angle 154.27^\circ$$

$$\mathbf{I}_1 = \frac{\Delta_1}{\Delta} = \underline{\mathbf{4.698\angle 95.24^\circ\ A}}$$

$$\mathbf{I}_2 = \frac{\Delta_2}{\Delta} = \underline{\mathbf{0.9928\angle 37.71^\circ\ A}}$$

Chapter 10, Problem 28.

In the circuit of Fig. 10.76, determine the mesh currents i_1 and i_2. Let $v_1 = 10\cos 4t$ **V** and $v_2 = 20\cos(4t - 30°)$V.

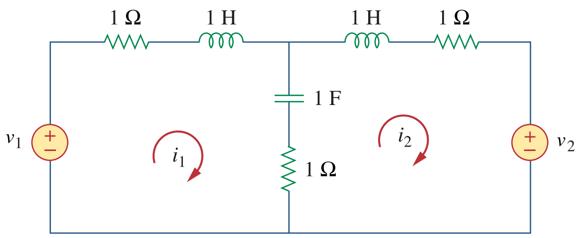


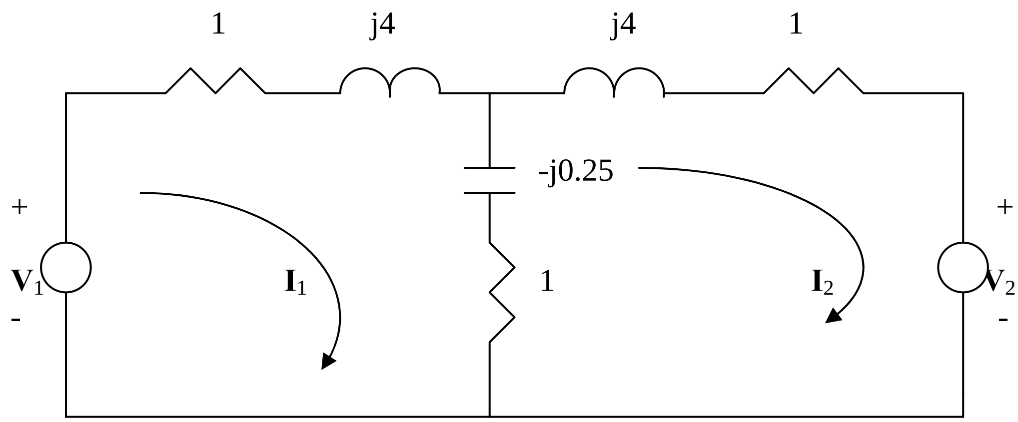
Figure 10.76
For Prob. 10.28.

Chapter 10, Solution 28.

$$1\text{H} \longrightarrow j\omega L = j4, \qquad 1\text{F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j1x4} = -j0.25$$

The frequency-domain version of the circuit is shown below, where

$$V_1 = 10\angle 0^o, \qquad V_2 = 20\angle -30^o \ .$$



$$V_1 = 10\angle 0^o, \qquad V_2 = 20\angle -30^o$$

Applying mesh analysis,

$$10 = (2 + j3.75)I_1 - (1 - j0.25)I_2 \qquad (1)$$

$$-20\angle -30^o = -(1 - j0.25)I_1 + (2 + j3.75)I_2 \qquad (2)$$

From (1) and (2), we obtain

$$\begin{pmatrix} 10 \\ -17.32 + j10 \end{pmatrix} = \begin{pmatrix} 2 + j3.75 & -1 + j0.25 \\ -1 + j0.25 & 2 + j3.75 \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \end{pmatrix}$$

Solving this leads to

$$I_1 = 2.741\angle -41.07^o, \qquad I_2 = 4.114\angle 92^o$$

Hence,

$i_1(t)$ = **$2.741\cos(4t - 41.07^\circ)$A**, $i_2(t)$ = **$4.114\cos(4t + 92^\circ)$A**.

Chapter 10, Problem 29.

By using mesh analysis, find $\mathbf{I}_1$ and $\mathbf{I}_2$ in the circuit depicted in Fig. 10.77.

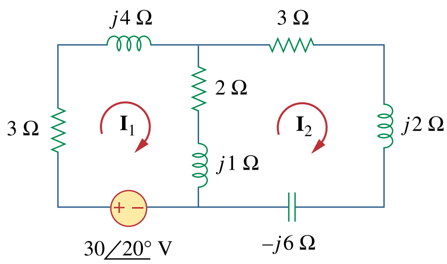


Figure 10.77
For Prob. 10.29.

Chapter 10, Solution 29.

For mesh 1,

$$(5+j5)\mathbf{I}_1 - (2+j)\mathbf{I}_2 - 30\angle 20° = 0$$

$$30\angle 20° = (5+j5)\mathbf{I}_1 - (2+j)\mathbf{I}_2 \qquad (1)$$

For mesh 2,

$$(5+j3-j6)\mathbf{I}_2 - (2+j)\mathbf{I}_1 = 0$$

$$0 = -(2+j)\mathbf{I}_1 + (5-j3)\mathbf{I}_2 \qquad (2)$$

From (1) and (2),

$$\begin{bmatrix} 30\angle 20° \\ 0 \end{bmatrix} = \begin{bmatrix} 5+j5 & -(2+j) \\ -(2+j) & 5-j3 \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix}$$

$$\Delta = 37 + j6 = 37.48\angle 9.21°$$

$$\Delta_1 = (30\angle 20°)(5.831\angle -30.96°) = 175\angle -10.96°$$

$$\Delta_2 = (30\angle 20°)(2.356\angle 26.56°) = 67.08\angle 46.56°$$

$$\mathbf{I}_1 = \frac{\Delta_1}{\Delta} = \underline{\mathbf{4.67\angle -20.17°\ A}}$$

$$\mathbf{I}_2 = \frac{\Delta_2}{\Delta} = \underline{\mathbf{1.79\angle 37.35°\ A}}$$

Chapter 10, Problem 30.

Use mesh analysis to find v_o in the circuit of Fig. 10.78. Let $v_{s1} = 120\cos(100t + 90°)$ V, $v_{s2} = 80\cos 100t$ V.

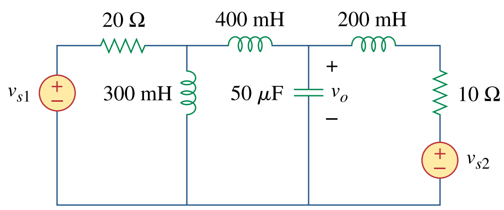
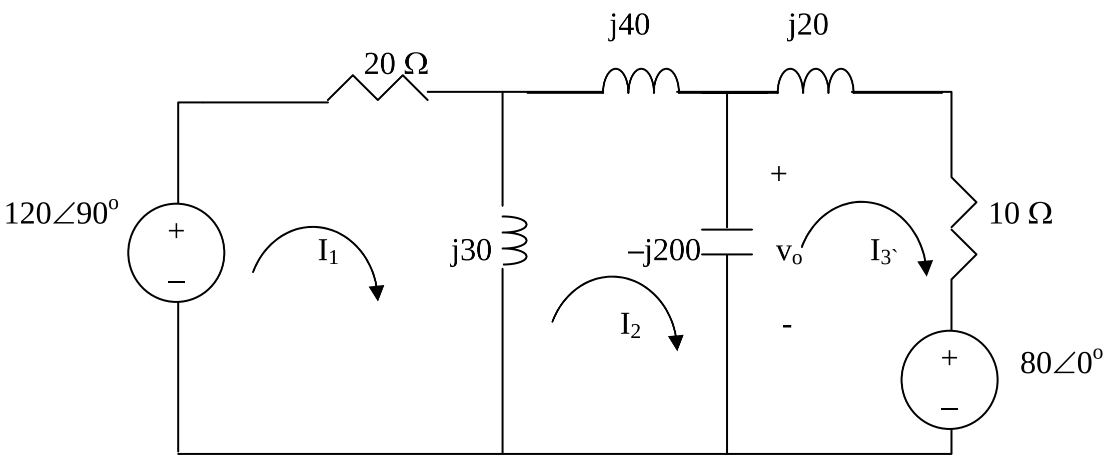


Figure 10.78
For Prob. 10.30.

Chapter 10, Solution 30.

$$300mH \longrightarrow j\omega L = j100x300x10^{-3} = j30$$
$$200mH \longrightarrow j\omega L = j100x200x10^{-3} = j20$$
$$400mH \longrightarrow j\omega L = j100x400x10^{-3} = j40$$
$$50\mu F \longrightarrow \frac{1}{j\omega C} = \frac{1}{j100x50x10^{-6}} = -j200$$

The circuit becomes that shown below.



For mesh 1,

$$-120 < 90^o + (20 + j30)I_1 - j30I_2 = 0 \longrightarrow j120 = (20 + j30)I_1 - j30I_2 \qquad (1)$$

For mesh 2,

$$-j30I_1 + (j30 + j40 - j200)I_2 + j200I_3 = 0 \longrightarrow 0 = -3I_1 - 13I_2 + 20I_3 \qquad (2)$$

For mesh 3,

$$80 + j200I_2 + (10 - j180)I_3 = 0 \rightarrow -8 = j20I_2 + (1 - j18)I_3 \qquad (3)$$

We put (1) to (3) in matrix form.

$$\begin{bmatrix} 2 + j3 & -j3 & 0 \\ -3 & -13 & 20 \\ 0 & j20 & 1 - j18 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} j12 \\ 0 \\ -8 \end{bmatrix}$$

This is an excellent candidate for MATLAB.

```
>> Z=[(2+3i),-3i,0;-3,-13,20;0,20i,(1-18i)]

Z =

   2.0000 + 3.0000i        0 - 3.0000i        0
  -3.0000           -13.0000           20.0000
        0                  0 +20.0000i   1.0000 -18.0000i

>> V=[12i;0;-8]

V =

        0 +12.0000i
        0
  -8.0000

>> I=inv(Z)*V

I =

   2.0557 + 3.5651i
   0.4324 + 2.1946i
   0.5894 + 1.9612i
```

$V_o = -j200(I_2 - I_3) = -j200(-0.157+j0.2334) = 46.68 + j31.4 = 56.26\angle 33.93^\circ$

v_o = **56.26cos(100t + 33.93˚ V**.

Chapter 10, Problem 31.

Use mesh analysis to determine current $\mathbf{I}_o$ in the circuit of Fig. 10.79 below.

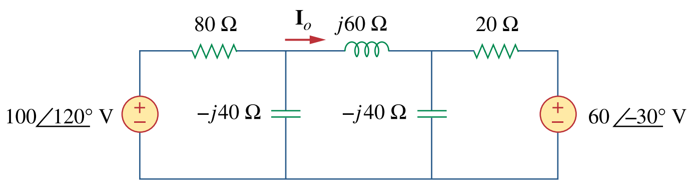
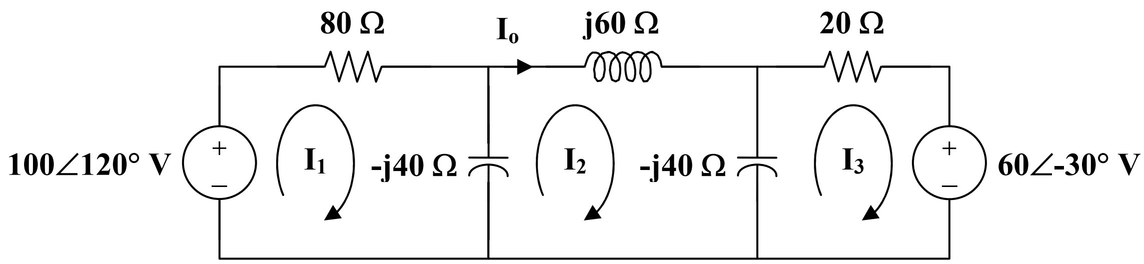


Figure 10.79
For Prob. 10.31.

Chapter 10, Solution 31.

Consider the network shown below.



For loop 1,

$$-100\angle 120° + (80 - j40)\mathbf{I}_1 + j40\mathbf{I}_2 = 0$$

$$10\angle 20° = 4(2 - j)\mathbf{I}_1 + j4\mathbf{I}_2 \qquad (1)$$

For loop 2,

$$j40\mathbf{I}_1 + (j60 - j80)\mathbf{I}_2 + j40\mathbf{I}_3 = 0$$

$$0 = 2\mathbf{I}_1 - \mathbf{I}_2 + 2\mathbf{I}_3 \qquad (2)$$

For loop 3,

$$60\angle -30° + (20 - j40)\mathbf{I}_3 + j40\mathbf{I}_2 = 0$$

$$-6\angle -30° = j4\mathbf{I}_2 + 2(1 - j2)\mathbf{I}_3 \qquad (3)$$

From (2),

$$2\mathbf{I}_3 = \mathbf{I}_2 - 2\mathbf{I}_1$$

Substituting this equation into (3),

$$-6\angle -30° = -2(1 - j2)\mathbf{I}_1 + (1 + j2)\mathbf{I}_2 \qquad (4)$$

From (1) and (4),

$$\begin{bmatrix} 10\angle 120° \\ -6\angle -30° \end{bmatrix} = \begin{bmatrix} 4(2 - j) & j4 \\ -2(1 - j2) & 1 + j2 \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 8 - j4 & -j4 \\ -2 + j4 & 1 + j2 \end{vmatrix} = 32 + j20 = 37.74\angle 32°$$

$$\Delta_2 = \begin{vmatrix} 8 - j4 & 10\angle 120° \\ -2 + j4 & -6\angle -30° \end{vmatrix} = -4.928 + j82.11 = 82.25\angle 93.44°$$

$$\mathbf{I}_o = \mathbf{I}_2 = \frac{\Delta_2}{\Delta} = \underline{\mathbf{2.179\angle 61.44°\ A}}$$

Chapter 10, Problem 32.

Determine $\mathbf{V}_o$ and $\mathbf{I}_o$ in the circuit of Fig. 10.80 using mesh analysis.

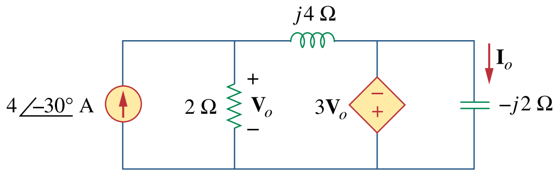
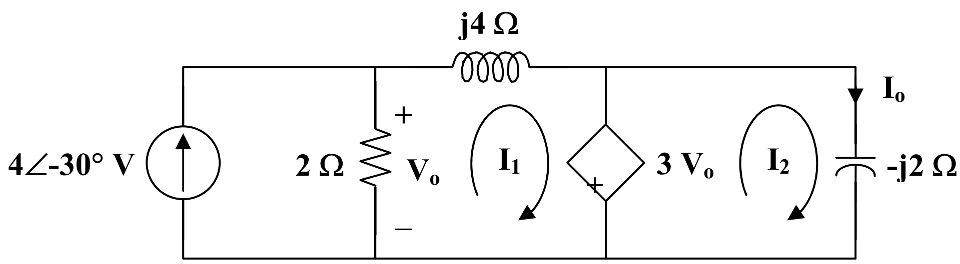


Figure 10.80
For Prob. 10.32.

Chapter 10, Solution 32.

Consider the circuit below.



For mesh 1,

$$(2+j4)\mathbf{I}_1 - 2(4\angle -30°) + 3\mathbf{V}_o = 0$$

where $$\mathbf{V}_o = 2(4\angle -30° - \mathbf{I}_1)$$

Hence,

$$(2+j4)\mathbf{I}_1 - 8\angle -30° + 6(4\angle -30° - \mathbf{I}_1) = 0$$

$$4\angle -30° = (1-j)\mathbf{I}_1$$

or $$\mathbf{I}_1 = 2\sqrt{2}\angle 15°$$

$$\mathbf{I}_o = \frac{3\mathbf{V}_o}{-j2} = \frac{3}{-j2}(2)(4\angle -30° - \mathbf{I}_1)$$

$$\mathbf{I}_o = j3(4\angle -30° - 2\sqrt{2}\angle 15°)$$

$$\mathbf{I}_o = \underline{\mathbf{8.485\angle 15°\ A}}$$

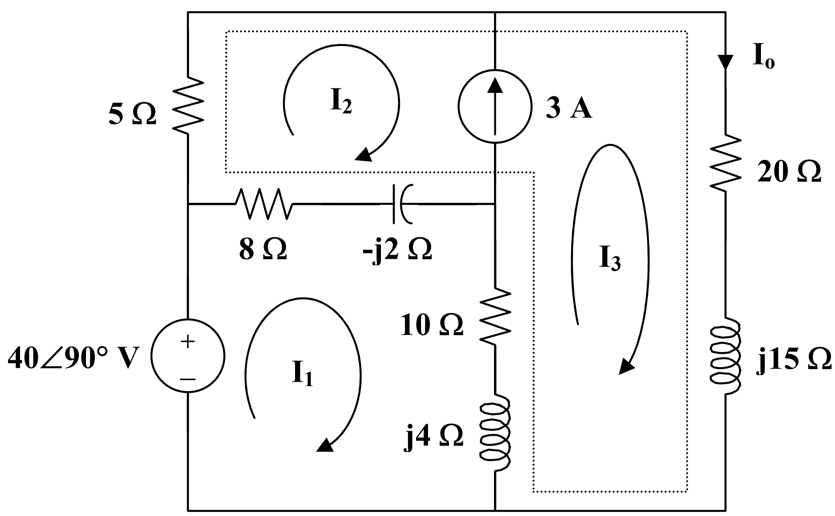
$$\mathbf{V}_o = \frac{-j2\mathbf{I}_o}{3} = \underline{\mathbf{5.657\angle -75°\ V}}$$

Chapter 10, Problem 34.

Use mesh analysis to find $\mathbf{I}_o$ in Fig. 10.28 (for Example 10.10).

Chapter 10, Solution 34.

The circuit is shown below.



For mesh 1,

$$-j40 + (18 + j2)\mathbf{I}_1 - (8 - j2)\mathbf{I}_2 - (10 + j4)\mathbf{I}_3 = 0 \qquad (1)$$

For the supermesh,

$$(13 - j2)\mathbf{I}_2 + (30 + j19)\mathbf{I}_3 - (18 + j2)\mathbf{I}_1 = 0 \qquad (2)$$

Also,

$$\mathbf{I}_2 = \mathbf{I}_3 - 3 \qquad (3)$$

Adding (1) and (2) and incorporating (3),

$$-j40 + 5(\mathbf{I}_3 - 3) + (20 + j15)\mathbf{I}_3 = 0$$

$$\mathbf{I}_3 = \frac{3 + j8}{5 + j3} = 1.465\angle 38.48°$$

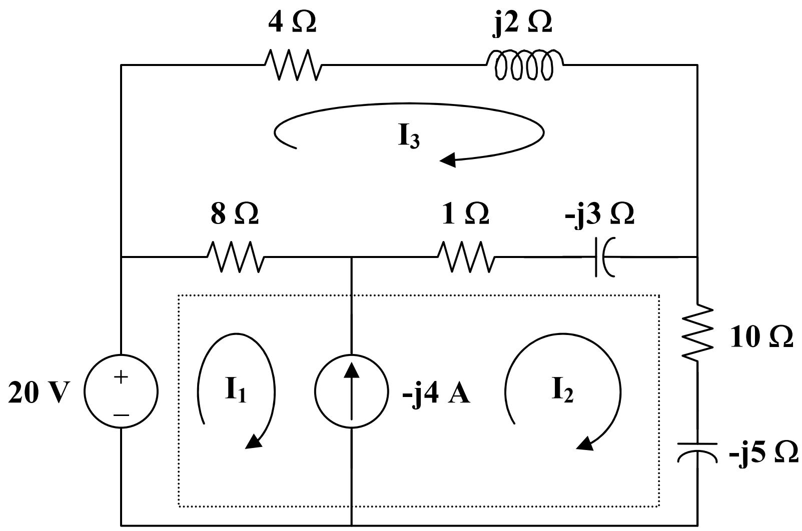
$$\mathbf{I}_o = \mathbf{I}_3 = \underline{\mathbf{1.465\angle 38.48°\ A}}$$

Chapter 10, Problem 35.

Calculate $\mathbf{I}_o$ in Fig. 10.30 (for Practice Prob. 10.10) using mesh analysis.

Chapter 10, Solution 35.

Consider the circuit shown below.



For the supermesh,

$$-20 + 8\mathbf{I}_1 + (11 - j8)\mathbf{I}_2 - (9 - j3)\mathbf{I}_3 = 0 \qquad (1)$$

Also,

$$\mathbf{I}_1 = \mathbf{I}_2 + j4 \qquad (2)$$

For mesh 3,

$$(13 - j)\mathbf{I}_3 - 8\mathbf{I}_1 - (1 - j3)\mathbf{I}_2 = 0 \qquad (3)$$

Substituting (2) into (1),

$$(19 - j8)\mathbf{I}_2 - (9 - j3)\mathbf{I}_3 = 20 - j32 \qquad (4)$$

Substituting (2) into (3),

$$-(9 - j3)\mathbf{I}_2 + (13 - j)\mathbf{I}_3 = j32 \qquad (5)$$

From (4) and (5),

$$\begin{bmatrix} 19 - j8 & -(9 - j3) \\ -(9 - j3) & 13 - j \end{bmatrix} \begin{bmatrix} \mathbf{I}_2 \\ \mathbf{I}_3 \end{bmatrix} = \begin{bmatrix} 20 - j32 \\ j32 \end{bmatrix}$$

$$\Delta = 167 - j69, \qquad \Delta_2 = 324 - j148$$

$$\mathbf{I}_2 = \frac{\Delta_2}{\Delta} = \frac{324 - j148}{167 - j69} = \frac{356.2\angle -24.55^\circ}{180.69\angle -22.45^\circ}$$

$$\mathbf{I}_2 = \underline{\mathbf{1.971\angle -2.1^\circ\ A}}$$

Chapter 10, Problem 36.

Compute $\mathbf{V}_o$ in the circuit of Fig. 10.81 using mesh analysis.

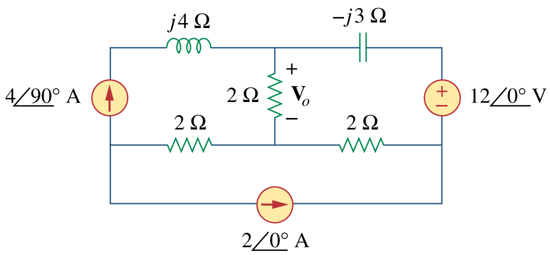
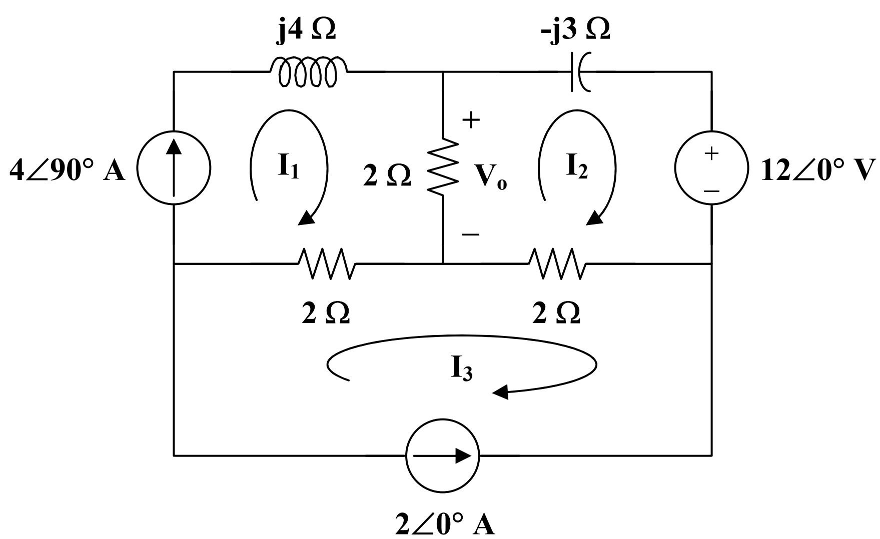


Figure 10.81
For Prob. 10.36.

Chapter 10, Solution 36.

Consider the circuit below.



Clearly,

$$\mathbf{I}_1 = 4\angle 90° = \text{j}4 \qquad \text{and} \qquad \mathbf{I}_3 = \text{-}2$$

For mesh 2,

$$(4 - \text{j}3)\mathbf{I}_2 - 2\mathbf{I}_1 - 2\mathbf{I}_3 + 12 = 0$$

$$(4 - \text{j}3)\mathbf{I}_2 - \text{j}8 + 4 + 12 = 0$$

$$\mathbf{I}_2 = \frac{\text{-}16 + \text{j}8}{4 - \text{j}3} = \text{-}3.52 - \text{j}0.64$$

Thus,

$$\mathbf{V}_o = 2(\mathbf{I}_1 - \mathbf{I}_2) = (2)(3.52 + \text{j}4.64) = 7.04 + \text{j}9.28$$

$$\mathbf{V}_o = \underline{\mathbf{11.648\angle 52.82°\ V}}$$

Chapter 10, Problem 37.

Use mesh analysis to find currents $\mathbf{I}_1$, $\mathbf{I}_2$, and $\mathbf{I}_3$ in the circuit of Fig. 10.82.

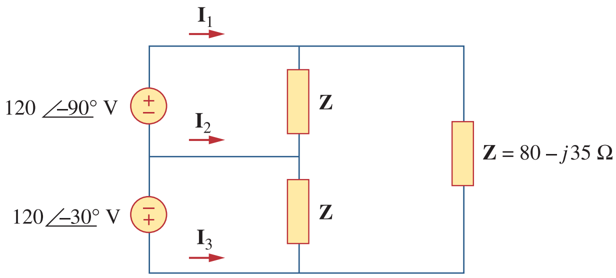
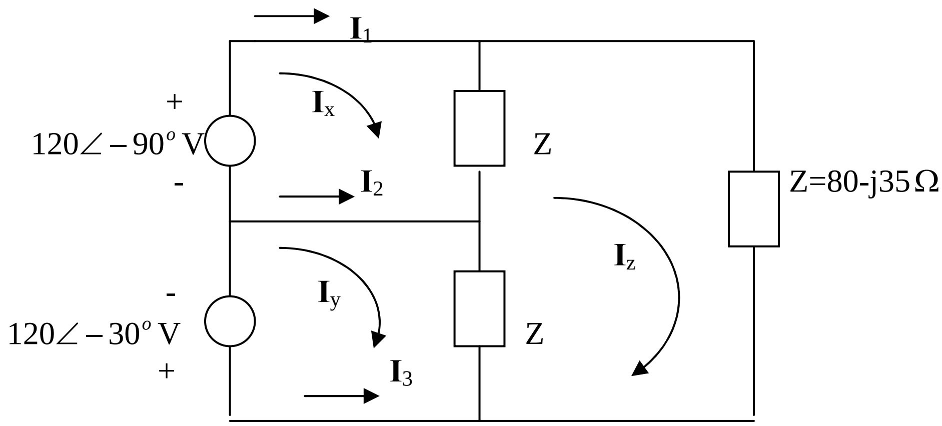


Figure 10.82
For Prob. 10.37.

Chapter 10, Solution 37.



For mesh x,

$$ZI_x - ZI_z = -j120 \qquad (1)$$

For mesh y,

$$ZI_y - ZI_z = -120\angle 30^o = -103.92 + j60 \qquad (2)$$

For mesh z,

$$-ZI_x - ZI_y + 3ZI_z = 0 \qquad (3)$$

Putting (1) to (3) together leads to the following matrix equation:

$$\begin{pmatrix} (80-j35) & 0 & (-80+j35) \\ 0 & (80-j35) & (-80+j35) \\ (-80+j35) & (-80+j35) & (240-j105) \end{pmatrix}\begin{pmatrix} I_x \\ I_y \\ I_z \end{pmatrix} = \begin{pmatrix} -j120 \\ -103.92+j60 \\ 0 \end{pmatrix} \longrightarrow \quad AI = B$$

Using MATLAB, we obtain

$$I = inv(A)*B = \begin{pmatrix} -0.2641 - j2.366 \\ -2.181 - j0.954 \\ -0.815 - j1.1066 \end{pmatrix}$$

$$I_1 = I_x = -0.2641 - j2.366 = \underline{2.38\angle -96.37^o} \text{ A}$$

$$I_2 = I_y - I_x = -1.9167 + j1.4116 = \underline{2.38\angle 143.63^o} \text{ A}$$

$$I_3 = -I_y = 2.181 + j0.954 = \underline{2.38\angle 23.63^o \text{ A}}$$

Chapter 10, Problem 38.

Using mesh analysis, obtain $\mathbf{I}_o$ in the circuit shown in Fig. 10.83.

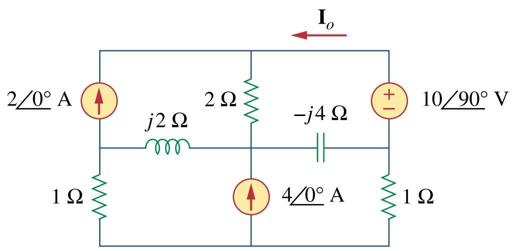
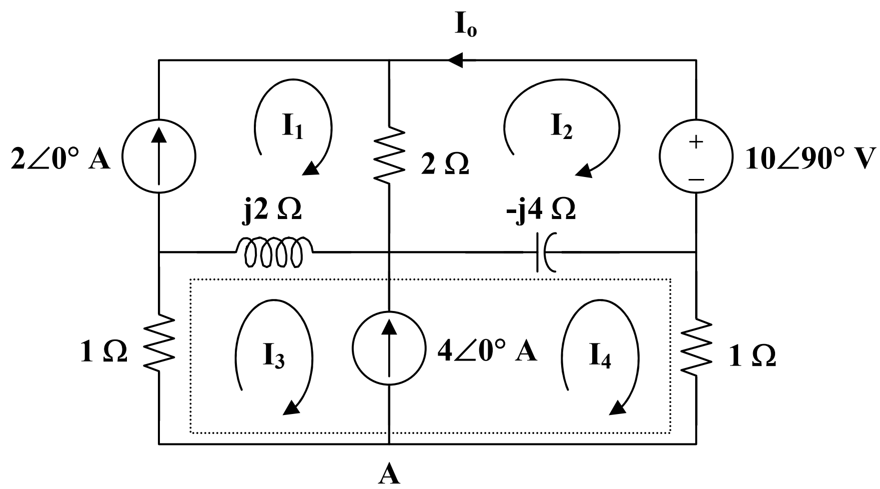


Figure 10.83
For Prob. 10.38.

Chapter 10, Solution 38.

Consider the circuit below.



Clearly,

$$\mathbf{I}_1 = 2 \qquad (1)$$

For mesh 2,

$$(2 - j4)\mathbf{I}_2 - 2\mathbf{I}_1 + j4\mathbf{I}_4 + 10\angle 90° = 0 \qquad (2)$$

Substitute (1) into (2) to get

$$(1 - j2)\mathbf{I}_2 + j2\mathbf{I}_4 = 2 - j5$$

For the supermesh,

$$(1 + j2)\mathbf{I}_3 - j2\mathbf{I}_1 + (1 - j4)\mathbf{I}_4 + j4\mathbf{I}_2 = 0$$

$$j4\mathbf{I}_2 + (1 + j2)\mathbf{I}_3 + (1 - j4)\mathbf{I}_4 = j4 \qquad (3)$$

At node A,

$$\mathbf{I}_3 = \mathbf{I}_4 - 4 \qquad (4)$$

Substituting (4) into (3) gives

$$j2\mathbf{I}_2 + (1 - j)\mathbf{I}_4 = 2(1 + j3) \qquad (5)$$

From (2) and (5),

$$\begin{bmatrix} 1 - j2 & j2 \\ j2 & 1 - j \end{bmatrix} \begin{bmatrix} \mathbf{I}_2 \\ \mathbf{I}_4 \end{bmatrix} = \begin{bmatrix} 2 - j5 \\ 2 + j6 \end{bmatrix}$$

$$\Delta = 3 - j3, \qquad \Delta_1 = 9 - j11$$

$$\mathbf{I}_o = \text{-}\mathbf{I}_2 = \frac{\text{-}\Delta_1}{\Delta} = \frac{\text{-}(9 - j11)}{3 - j3} = \frac{1}{3}(\text{-}10 + j)$$

$$\mathbf{I}_o = \underline{\mathbf{3.35\angle 174.3° \ A}}$$

Chapter 10, Problem 39.

Find $\mathbf{I}_1$, $\mathbf{I}_2$, $\mathbf{I}_3$, and $\mathbf{I}_x$ in the circuit of Fig. 10.84.

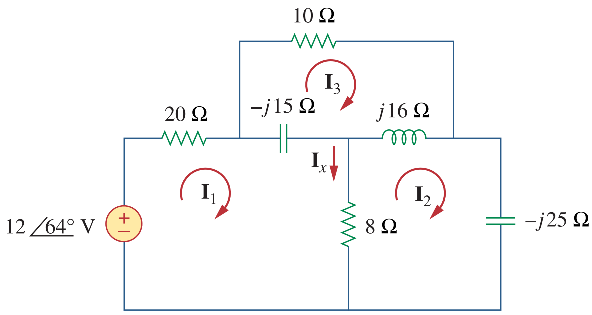


Figure 10.84
For Prob. 10.39.

Chapter 10, Solution 39.

For mesh 1,

$$(28 - j15)I_1 - 8I_2 + j15I_3 = 12\angle 64^o \qquad (1)$$

For mesh 2,

$$-8I_1 + (8 - j9)I_2 - j16I_3 = 0 \qquad (2)$$

For mesh 3,

$$j15I_1 - j16I_2 + (10 + j)I_3 = 0 \qquad (3)$$

In matrix form, (1) to (3) can be cast as

$$\begin{pmatrix} (28-j15) & -8 & j15 \\ -8 & (8-j9) & -j16 \\ j15 & -j16 & (10+j) \end{pmatrix}\begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix} = \begin{pmatrix} 12\angle 64^o \\ 0 \\ 0 \end{pmatrix} \quad \text{or} \quad AI = B$$

Using MATLAB,

I = inv(A)*B

$$I_1 = -0.128 + j0.3593 = \underline{0.3814\angle 109.6^o \text{ A}}$$

$$I_2 = -0.1946 + j0.2841 = \underline{0.3443\angle 124.4^o \text{ A}}$$

$$I_3 = 0.0718 - j0.1265 = \underline{0.1455\angle -60.42^o \text{ A}}$$

$$I_x = I_1 - I_2 = 0.0666 + j0.0752 = \underline{0.1005\angle 48.5^o \text{ A}}$$

Chapter 10, Problem 40.

Find i_o in the circuit shown in Fig. 10.85 using superposition.

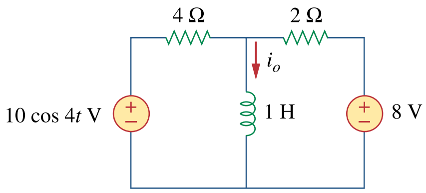
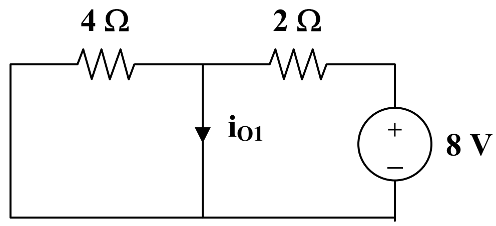


Figure 10.85
For Prob. 10.40.

Chapter 10, Solution 40.

Let $i_O = i_{O1} + i_{O2}$, where i_{O1} is due to the dc source and i_{O2} is due to the ac source. For i_{O1}, consider the circuit in Fig. (a).

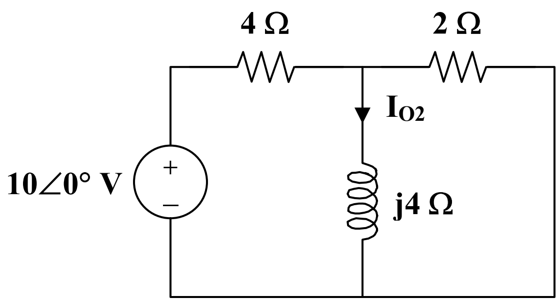


(a)

Clearly,

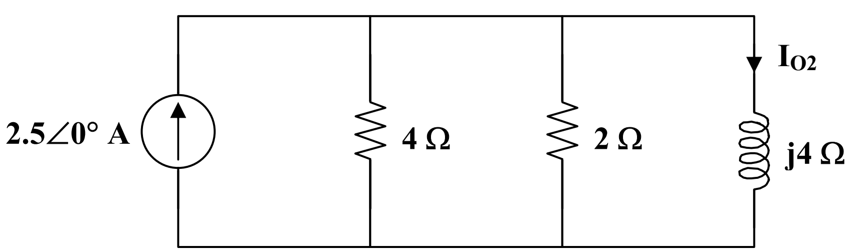
$$i_{O1} = 8/2 = 4 \text{ A}$$

For i_{O2}, consider the circuit in Fig. (b).



(b)

If we transcribe the voltage source, we have the circuit in Fig. (c), where $4 \parallel 2 = 4/3\,\Omega$.



(c)

By the current division principle,

$$\mathbf{I}_{O2} = \frac{4/3}{4/3 + j4}(2.5\angle 0°)$$

$$\mathbf{I}_{O2} = 0.25 - j0.75 = 0.79\angle -71.56°$$

Thus, $\quad i_{O2} = 0.79\cos(4t - 71.56°) \text{ A}$

Therefore,

$$i_O = i_{O1} + i_{O2} = \underline{\mathbf{4 + 0.79\cos(4t - 71.56°)\ A}}$$

Chapter 10, Problem 41.

Find v_o for the circuit in Fig. 10.86, assuming that $v_s = 6\cos 2t + 4\sin 4t$ V.

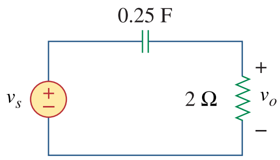


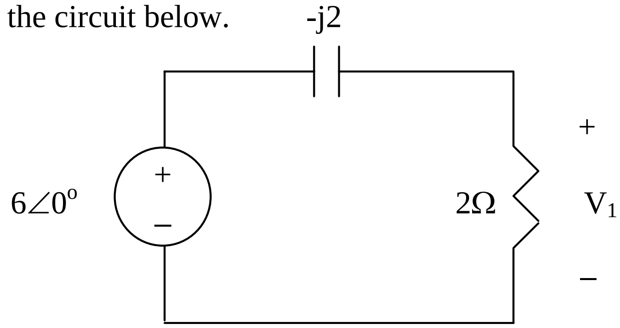
Figure 10.86
For Prob. 10.41.

Chapter 10, Solution 41.

We apply superposition principle. We let

$$v_o = v_1 + v_2$$

where v_1 and v_2 are due to the sources 6cos2t and 4sin4t respectively. To find v_1, consider the circuit below.



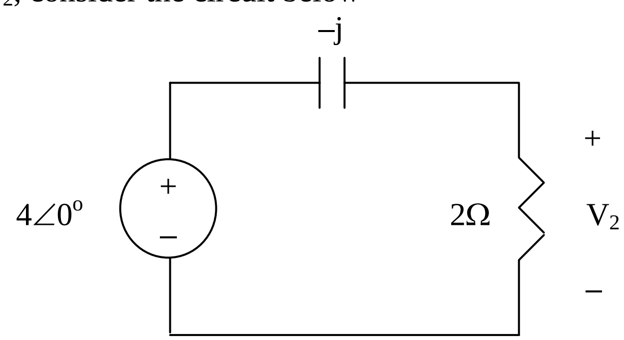
$$1/4F \longrightarrow \frac{1}{j\omega C} = \frac{1}{j2x1/4} = -j2$$

$$V_1 = \frac{2}{2-j2}(6) = 3 + j3 = 4.2426 < 45^o$$

Thus,

$$v_1 = 4.2426\cos(2t + 45^o)$$

To get v_2, consider the circuit below



$$1/4F \longrightarrow \frac{1}{j\omega C} = \frac{1}{j4x1/4} = -j1$$

$$V_2 = \frac{2}{2-j}(4) = 3.2 + j11.6 = 3.578 < 26.56^o$$

$$v_2 = 3.578\sin(4t + 26.56^o)$$

Hence,

v_o = **<u>4.243cos(2t + 45°) + 3.578sin(4t + 25.56°) V</u>**.

Chapter 10, Problem 42.

Solve for $\mathbf{I}_o$ in the circuit of Fig. 10.87.

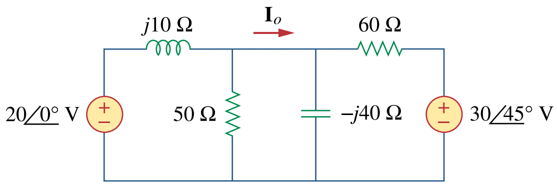
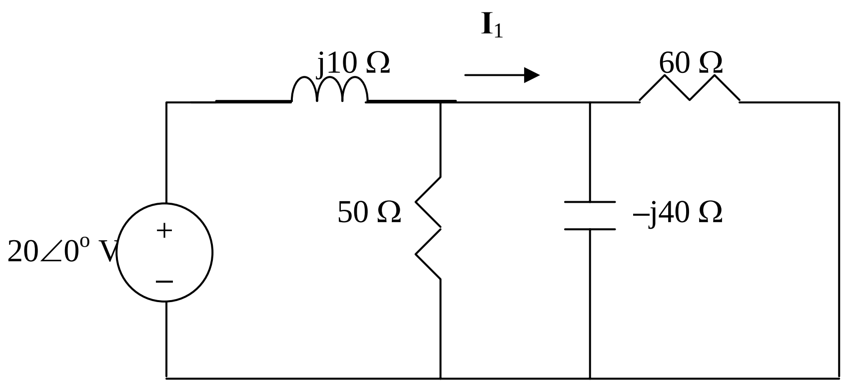


Figure 10.87
For Prob. 10.42.

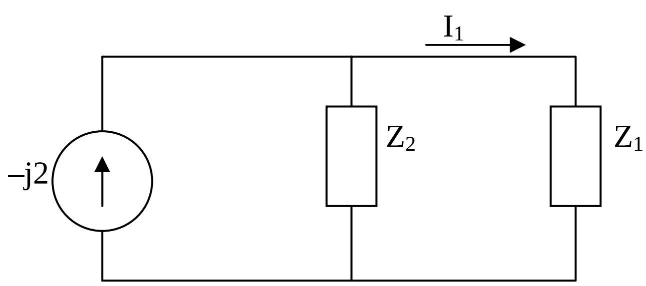
Chapter 10, Solution 42.

Let $I_o = I_1 + I_2$

where I_1 and I_2 are due to $20<0^o$ and $30<45^o$ sources respectively. To get I_1, we use the circuit below.



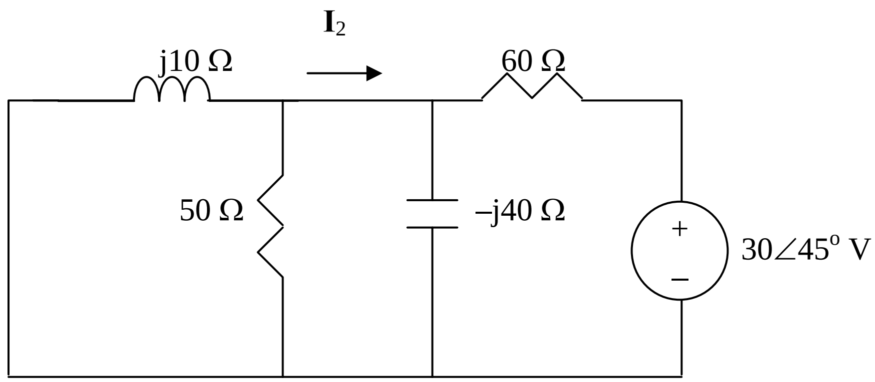
Let $Z_1 = -j40//60 = 18.4615 - j27.6927$, $Z_2 = j10//50 = 1.9231 + j9.615$
Transforming the voltage source to a current source leads to the circuit below.



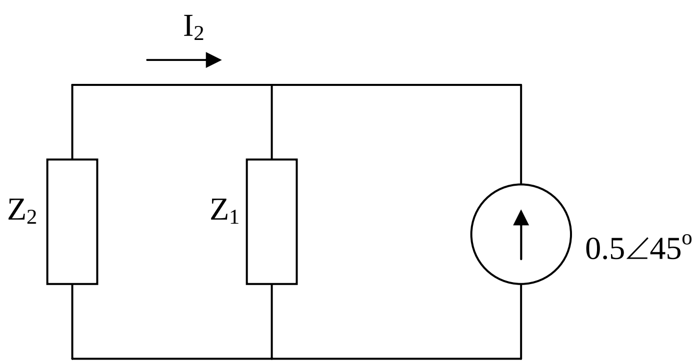
Using current division,

$$I_1 = \frac{Z_2}{Z_1 + Z_2}(-j2) = 0.6217 + j0.3626$$

To get I_2, we use the circuit below.



After transforming the voltage source, we obtain the circuit below.



Using current division,

$$I_2 = \frac{-Z_1}{Z_1 + Z_2}(0.5 < 45^o) = -0.5275 - j0.3077$$

Hence,

$$I_o = I_1 + I_2 = 0.0942 + j0.0509 = \underline{0.109 < 30^o \text{ A}}$$

Chapter 10, Problem 43.

Using the superposition principle, find i_x in the circuit of Fig. 10.88.

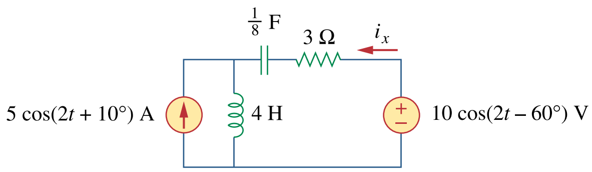


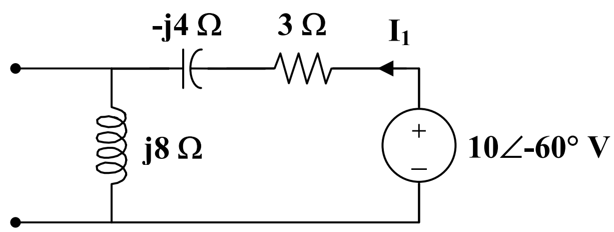
Figure 10.88
For Prob. 10.43.

Chapter 10, Solution 43.

Let $\mathbf{I}_x = \mathbf{I}_1 + \mathbf{I}_2$, where $\mathbf{I}_1$ is due to the voltage source and $\mathbf{I}_2$ is due to the current source.

$$\omega = 2$$
$$5\cos(2t + 10°) \longrightarrow 5\angle 10°$$
$$10\cos(2t - 60°) \longrightarrow 10\angle\text{-}60°$$
$$4\text{ H} \longrightarrow j\omega L = j8$$
$$\frac{1}{8}\text{ F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(2)(1/8)} = \text{-}j4$$

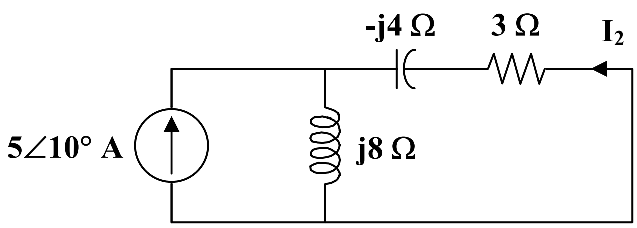
For $\mathbf{I}_1$, consider the circuit in Fig. (a).



(a)

$$\mathbf{I}_1 = \frac{10\angle\text{-}60°}{3 + j8 - j4} = \frac{10\angle\text{-}60°}{3 + j4}$$

For $\mathbf{I}_2$, consider the circuit in Fig. (b).



(b)

$$\mathbf{I}_2 = \frac{\text{-}j8}{3 + j8 - j4}(5\angle 10°) = \frac{\text{-}j40\angle 10°}{3 + j4}$$

$$\mathbf{I}_x = \mathbf{I}_1 + \mathbf{I}_2 = \frac{1}{3 + j4}(10\angle\text{-}60° - j40\angle 10°)$$
$$\mathbf{I}_x = \frac{49.51\angle\text{-}76.04°}{5\angle 53.13°} = 9.902\angle\text{-}129.17°$$

Therefore, $i_x =$ **9.902 cos(2t – 129.17°) A**

Chapter 10, Problem 44.

Use the superposition principle to obtain v_x in the circuit of Fig. 10.89. Let $v_s = 50\sin 2t$ V and $i_s = 12\cos(6t + 10°)$A.

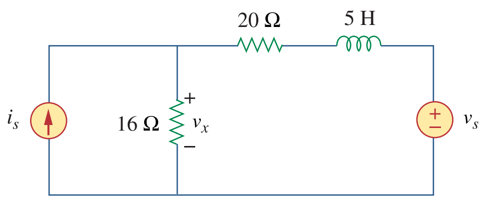


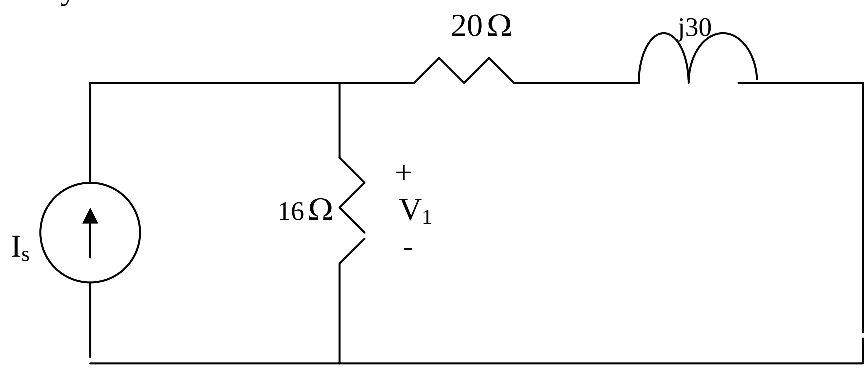
Figure 10.89
For Prob. 10.44.

Chapter 10, Solution 44.

Let $v_x = v_1 + v_2$, where v_1 and v_2 are due to the current source and voltage source respectively.

For v_1, $\omega = 6$, $5\,H \longrightarrow j\omega L = j30$

The frequency-domain circuit is shown below.

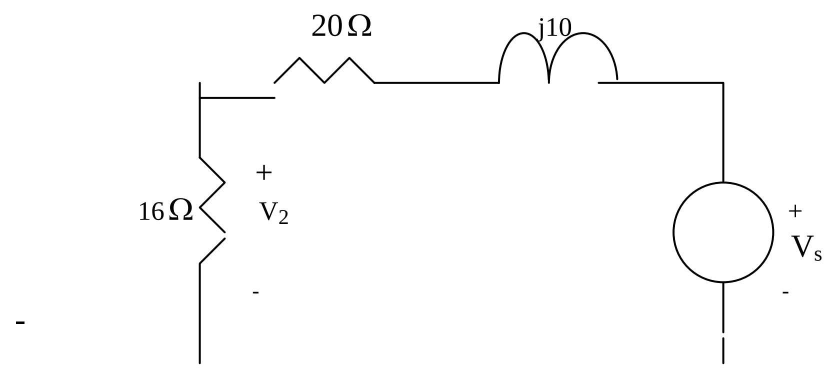


Let $Z = 16//(20 + j30) = \dfrac{16(20+j30)}{36+j30} = 11.8 + j3.497 = 12.31\angle 16.5^{o}$

$V_1 = I_s Z = (12\angle 10^{o})(12.31\angle 16.5^{o}) = 147.7\angle 26.5^{o} \longrightarrow v_1 = 147.7\cos(6t + 26.5^{o})$ V

For v_2, $\omega = 2$, $5\,H \longrightarrow j\omega L = j10$

The frequency-domain circuit is shown below.



Using voltage division,

$$V_2 = \frac{16}{16+20+j10} V_s = \frac{16(50\angle 0^{o})}{36+j10} = 21.41\angle -15.52^{o} \longrightarrow v_2 = 21.41\sin(2t - 15.52^{o})\text{ V}$$

Thus,

$$v_x = \underline{147.7\cos(6t + 26.5^{o}) + 21.41\sin(2t - 15.52^{o})\text{ V}}$$

Chapter 10, Problem 45.

Use superposition to find $i(t)$ in the circuit of Fig. 10.90.

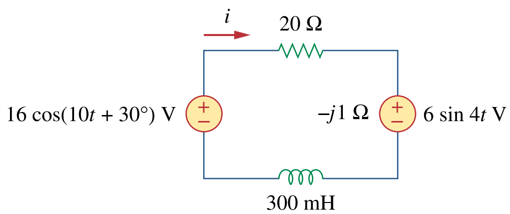
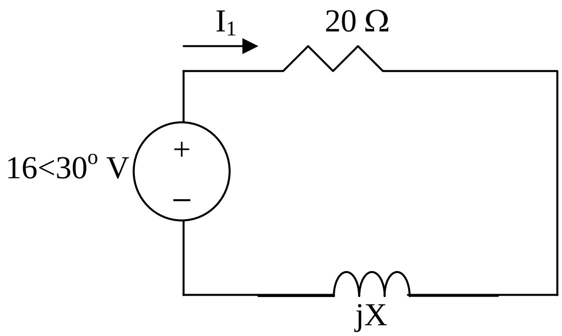


Figure 10.90
For Prob. 10.45.

Chapter 10, Solution 45.

Let $i = i_1 + i_2$, where i_1 and i_2 are due to 16cos(10t +30°) and 6sin4t sources respectively. To find i_1, consider the circuit below.

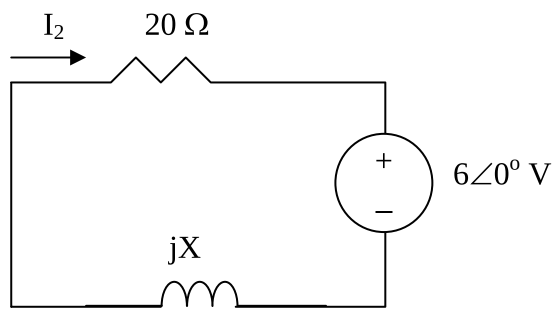


$$X = \omega L = 10x300x10^{-3} = 3$$

$$I_1 = \frac{16 < 30^o}{20 + j3} = 0.7911$$

$$i_1 = 0.7911\cos(10t + 21.47^o)\text{ A}$$

To find i_2, consider the circuit below.



$$X = \omega L = 4x300x10^{-3} = 1.2$$

$$I_2 = -\frac{6 < 0^o}{20 + j1.2} = 0.2995 < 176.6^o$$

$$i_1 = 0.2995\sin(4t + 176.6^o)\text{ A}$$

Thus,

$$i = i_1 + i_2 = \underline{0.7911\cos(10t + 21.47^o) + 0.2995\sin(4t + 176.6^o)\text{ A}}$$

= **791.1cos(10t+21.47°)+299.5sin(4t+176.6°) mA**

Chapter 10, Problem 46.
Solve for $v_o(t)$ in the circuit of Fig. 10.91 using the superposition principle.

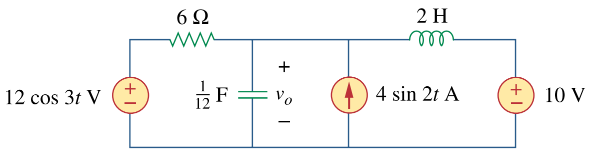
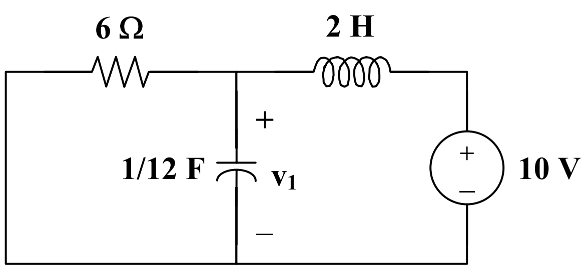


Figure 10.91
For Prob. 10.46.

Chapter 10, Solution 46.

Let $v_o = v_1 + v_2 + v_3$, where v_1, v_2, and v_3 are respectively due to the 10-V dc source, the ac current source, and the ac voltage source. For v_1 consider the circuit in Fig. (a).



(a)

The capacitor is open to dc, while the inductor is a short circuit. Hence,

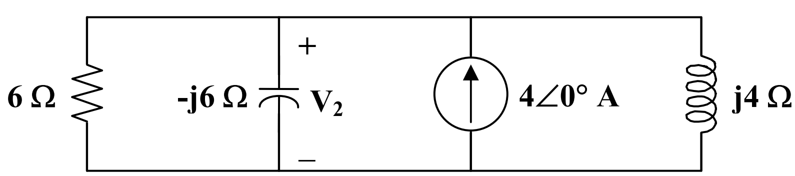
$$v_1 = 10 \text{ V}$$

For v_2, consider the circuit in Fig. (b).

$$\omega = 2$$

$$2 \text{ H} \longrightarrow j\omega L = j4$$

$$\frac{1}{12}\text{ F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(2)(1/12)} = \text{-}j6$$



(b)

Applying nodal analysis,

$$4 = \frac{\mathbf{V}_2}{6} + \frac{\mathbf{V}_2}{-\text{j}6} + \frac{\mathbf{V}_2}{\text{j}4} = \left(\frac{1}{6} + \frac{\text{j}}{6} - \frac{\text{j}}{4}\right)\mathbf{V}_2$$

$$\mathbf{V}_2 = \frac{24}{1 - \text{j}0.5} = 21.45\angle 26.56°$$

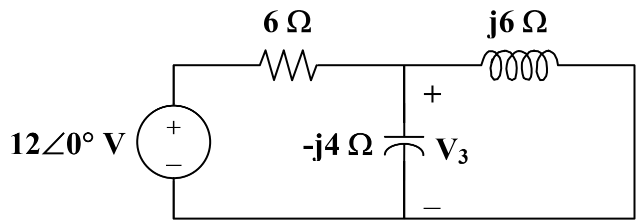
Hence, $\quad v_2 = 21.45\sin(2t + 26.56°)\ \text{V}$

For v_3, consider the circuit in Fig. (c).

$$\omega = 3$$

$$2\ \text{H} \longrightarrow \text{j}\omega\text{L} = \text{j}6$$

$$\frac{1}{12}\ \text{F} \longrightarrow \frac{1}{\text{j}\omega\text{C}} = \frac{1}{\text{j}(3)(1/12)} = -\text{j}4$$



(c)

At the non-reference node,

$$\frac{12 - \mathbf{V}_3}{6} = \frac{\mathbf{V}_3}{-\text{j}4} + \frac{\mathbf{V}_3}{\text{j}6}$$

$$\mathbf{V}_3 = \frac{12}{1 + \text{j}0.5} = 10.73\angle -26.56°$$

Hence, $\quad v_3 = 10.73\cos(3t - 26.56°)\ \text{V}$

Therefore, $\quad v_o =$ **<u>10 + 21.45 sin(2t + 26.56°) + 10.73 cos(3t – 26.56°) V</u>**

Chapter 10, Problem 47.

Determine i_o in the circuit of Fig. 10.92, using the superposition principle.

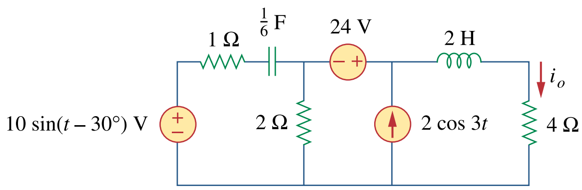
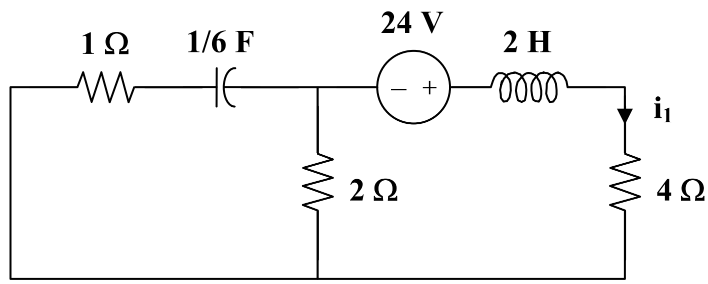


Figure 10.92
For Prob. 10.47.

Chapter 10, Solution 47.

Let $i_o = i_1 + i_2 + i_3$, where i_1, i_2, and i_3 are respectively due to the 24-V dc source, the ac voltage source, and the ac current source. For i_1, consider the circuit in Fig. (a).

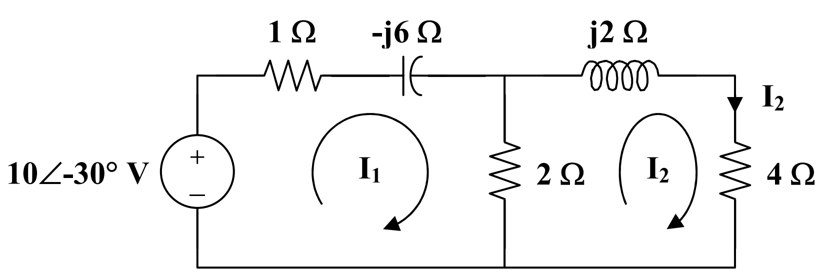


Since the capacitor is an open circuit to dc,

$$i_1 = \frac{24}{4+2} = 4 \text{ A}$$

For i_2, consider the circuit in Fig. (b).

$$\omega = 1$$
$$2 \text{ H} \longrightarrow j\omega L = j2$$
$$\frac{1}{6} \text{ F} \longrightarrow \frac{1}{j\omega C} = \text{-}j6$$



(b)

For mesh 1,

$$\text{-}10\angle\text{-}30° + (3 - j6)\mathbf{I}_1 - 2\mathbf{I}_2 = 0$$
$$10\angle\text{-}30° = 3(1 - 2j)\mathbf{I}_1 - 2\mathbf{I}_2 \qquad (1)$$

For mesh 2,

$$0 = \text{-}2\mathbf{I}_1 + (6 + j2)\mathbf{I}_2$$
$$\mathbf{I}_1 = (3 + j)\mathbf{I}_2 \qquad (2)$$

Chapter 10, Problem 48.

Find i_o in the circuit of Fig. 10.93 using superposition.

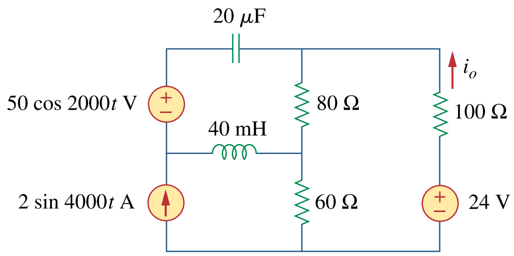


Figure 10.93
For Prob. 10.48.

Chapter 10, Solution 48.

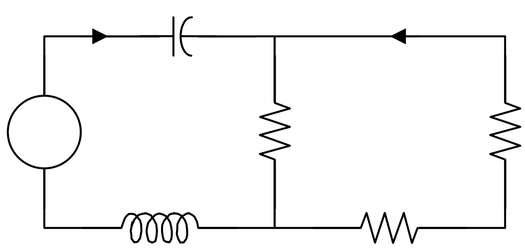
Let $i_O = i_{O1} + i_{O2} + i_{O3}$, where i_{O1} is due to the ac voltage source, i_{O2} is due to the dc voltage source, and i_{O3} is due to the ac current source. For i_{O1}, consider the circuit in Fig. (a).

$$\omega = 2000$$

$$50\cos(2000t) \longrightarrow 50\angle 0°$$

$$40\text{ mH} \longrightarrow j\omega L = j(2000)(40\times 10^{-3}) = j80$$

$$20\ \mu\text{F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(2000)(20\times 10^{-6})} = -j25$$



$$80\,||\,(60+100) = 160/3$$

$$\mathbf{I} = \frac{50}{160/3 + j80 - j25} = \frac{30}{32 + j33}$$

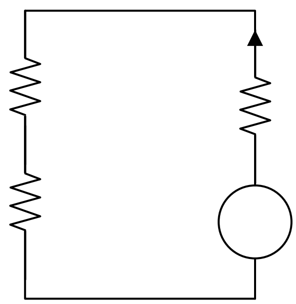
Using current division,

$$\mathbf{I}_{O1} = \frac{-80\,\mathrm{I}}{80+160} = \frac{-1}{3}\mathbf{I} = \frac{10\angle 180°}{46\angle 45.9°}$$

$$\mathbf{I}_{O1} = 0.217\angle 134.1°$$

Hence, $$i_{O1} = 0.217\cos(2000t + 134.1°) \text{ A}$$

For i_{O2}, consider the circuit in Fig. (b).



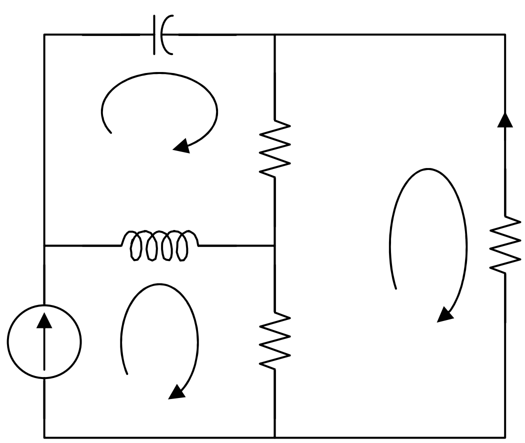
$$i_{O2} = \frac{24}{80+60+100} = 0.1 \text{ A}$$

For i_{O3}, consider the circuit in Fig. (c).

$$\omega = 4000$$

$$2\cos(4000t) \longrightarrow 2\angle 0°$$

$$40 \text{ mH} \longrightarrow j\omega L = j(4000)(40\times 10^{-3}) = j160$$



$$20\ \mu\text{F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(4000)(20\times 10^{-6})} = -j12.5$$

For mesh 1,

$$\mathbf{I}_1 = 2 \qquad (1)$$

For mesh 2,

$$(80 + j160 - j12.5)\mathbf{I}_2 - j160\mathbf{I}_1 - 80\mathbf{I}_3 = 0$$

Simplifying and substituting (1) into this equation yields

$$(8 + j14.75)\mathbf{I}_2 - 8\mathbf{I}_3 = j32 \qquad (2)$$

For mesh 3,

$$240\mathbf{I}_3 - 60\mathbf{I}_1 - 80\mathbf{I}_2 = 0$$

Simplifying and substituting (1) into this equation yields

$$\mathbf{I}_2 = 3\mathbf{I}_3 - 1.5 \qquad (3)$$

Substituting (3) into (2) yields

$$(16 + j44.25)\mathbf{I}_3 = 12 + j54.125$$

$$\mathbf{I}_3 = \frac{12 + j54.125}{16 + j44.25} = 1.1782\angle 7.38°$$

$$\mathbf{I}_{O3} = -\mathbf{I}_3 = -1.1782\angle 7.38°$$

Hence, $\quad i_{O3} = -1.1782\sin(4000t + 7.38°)$ A

Therefore, $\quad i_O =$ **<u>0.1 + 0.217 cos(2000t + 134.1°) – 1.1782 sin(4000t + 7.38°) A</u>**

Chapter 10, Problem 49.

Using source transformation, find *i* in the circuit of Fig. 10.94.

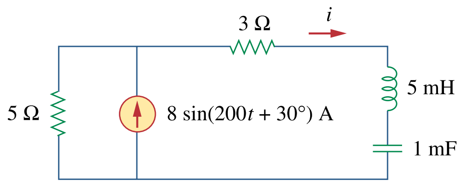


Figure 10.94
For Prob. 10.49.

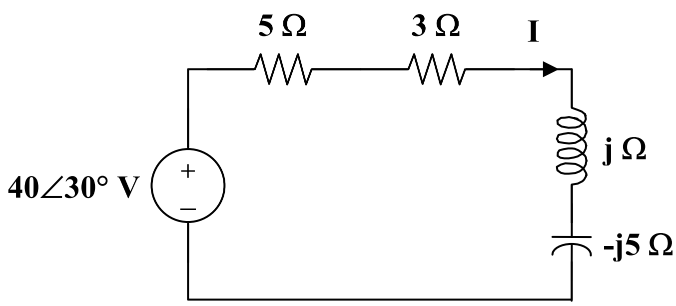
Chapter 10, Solution 49.

$$8\sin(200t + 30°) \longrightarrow 8\angle 30°, \quad \omega = 200$$

$$5\text{ mH} \longrightarrow j\omega L = j(200)(5 \times 10^{-3}) = j$$

$$1\text{ mF} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(200)(1 \times 10^{-3})} = -j5$$

After transforming the current source, the circuit becomes that shown in the figure below.



$$\mathbf{I} = \frac{40\angle 30°}{5 + 3 + j - j5} = \frac{40\angle 30°}{8 - j4} = 4.472\angle 56.56°$$

$$i = \underline{\mathbf{4.472\ sin(200t + 56.56°)\ A}}$$

Chapter 10, Problem 50.

Use source transformation to find v_o in the circuit of Fig. 10.95.

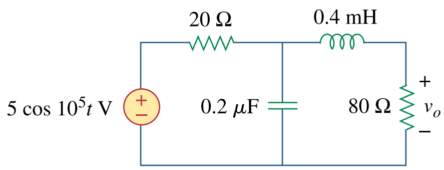


Figure 10.95
For Prob. 10.50.

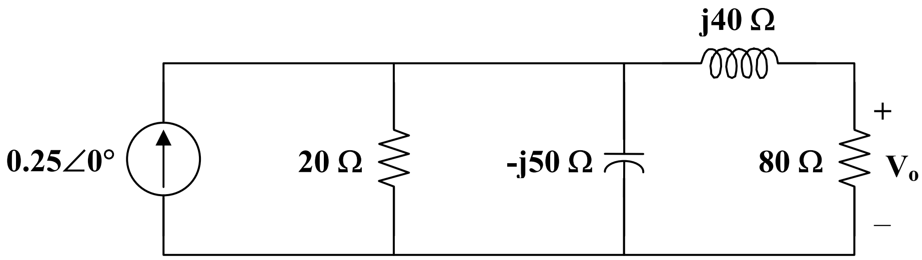
Chapter 10, Solution 50.

$$5\cos(10^5 t) \longrightarrow 5\angle 0°, \quad \omega = 10^5$$

$$0.4 \text{ mH} \longrightarrow j\omega L = j(10^5)(0.4\times10^{-3}) = j40$$

$$0.2\ \mu\text{F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(10^5)(0.2\times10^{-6})} = -j50$$

After transforming the voltage source, we get the circuit in Fig. (a).

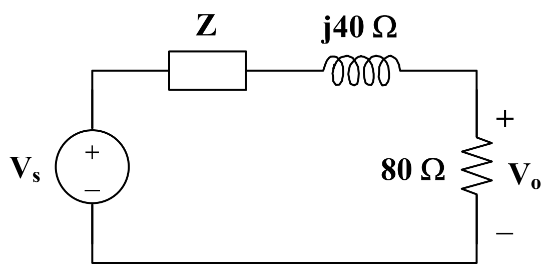


(a)

Let $\quad \mathbf{Z} = 20 \,||\, -j50 = \dfrac{-j100}{2 - j5}$

and $\quad \mathbf{V}_s = (0.25\angle 0°)\,\mathbf{Z} = \dfrac{-j25}{2 - j5}$

With these, the current source is transformed to obtain the circuit in Fig.(b).



(b)

By voltage division,

$$\mathbf{V}_o = \frac{80}{\mathbf{Z} + 80 + j40}\mathbf{V}_s = \frac{80}{\dfrac{-j100}{2 - j5} + 80 + j40} \cdot \frac{-j25}{2 - j5}$$

$$\mathbf{V}_o = \frac{8(-j25)}{36 - j42} = 3.615\angle -40.6°$$

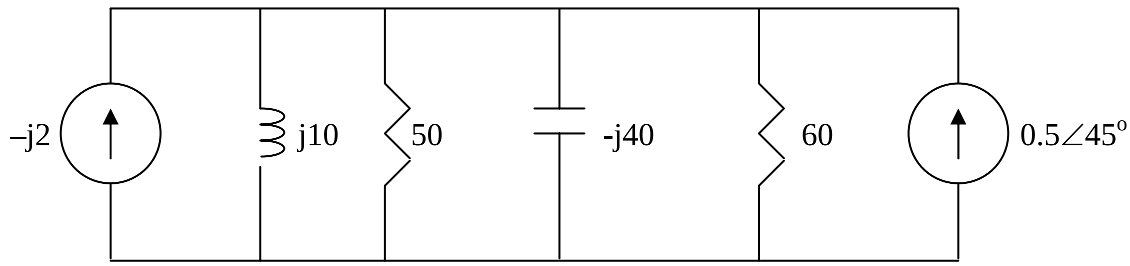
Therefore, $\quad v_o = \underline{\mathbf{3.615 \cos(10^5 t - 40.6°) \text{ V}}}$

Chapter 10, Problem 51.

Use source transformation to find $\mathbf{I}_o$ in the circuit of Prob. 10.42.

Chapter 10, Solution 51.

Transforming the voltage sources into current sources, we have the circuit as shown below.



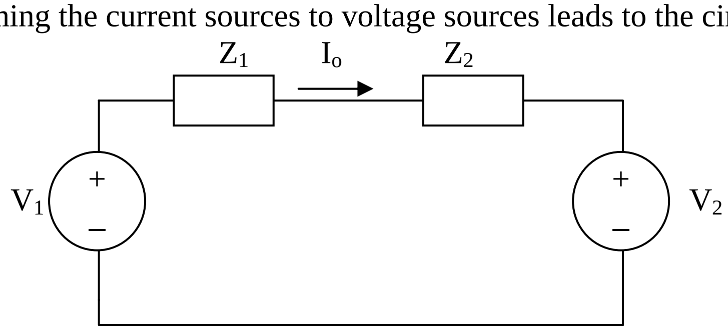
Let $Z_1 = j10//50 = \dfrac{j10x50}{50+j10} = 1.9231 + j9.615$

$V_1 = -j2Z_1 = 19.231 - j3.846$

Let $Z_2 = -j40//60 = \dfrac{-j40x60}{60-j40} = 18.4615 - j27.6923$

$V_2 = Z_2 x0.5 < 45^o = 16.315 - 3.263$

Transforming the current sources to voltage sources leads to the circuit below.



Applying KVL to the loop gives

$$-V_1 + I_o(Z_1+Z_2) + V_2 = 0 \longrightarrow I_o = \frac{V_1 - V_2}{Z_1+Z_2}$$

$$I_o = \frac{19.231 - j3.846 - 16.316 + j3.263}{1.9231 + j9.615 + 18.4615 - j27.6923} = \underline{0.1093 < 30^o\ \text{A}} = \underline{\mathbf{109.3\angle 30^\circ\ mA}}$$

Chapter 10, Problem 52.

Use the method of source transformation to find $\mathbf{I}_x$ in the circuit of Fig. 10.96.

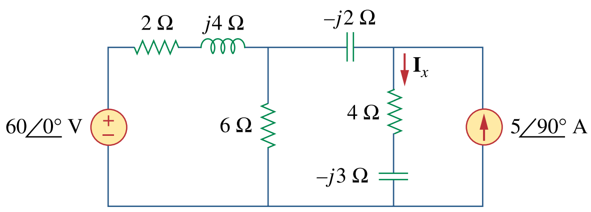


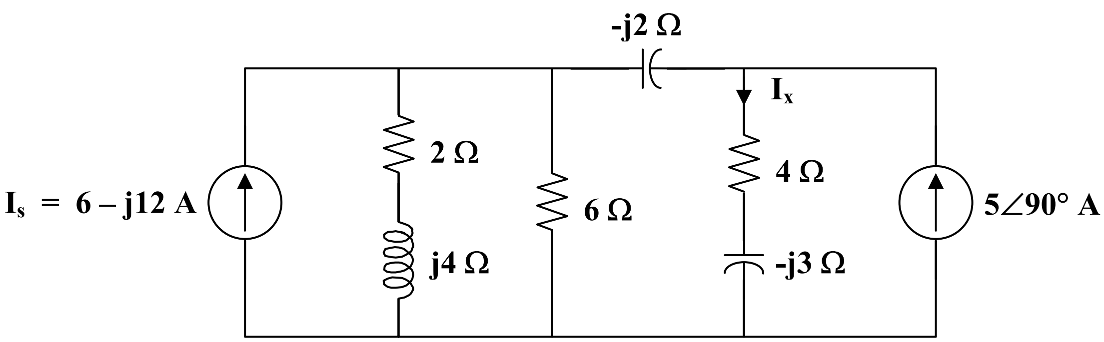
Figure 10.96
For Prob. 10.52.

Chapter 10, Solution 52.

We transform the voltage source to a current source.

$$\mathbf{I}_s = \frac{60\angle 0°}{2 + j4} = 6 - j12$$

The new circuit is shown in Fig. (a).

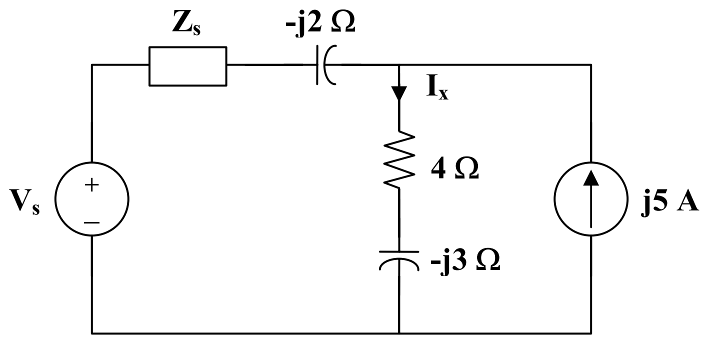


(a)

Let
$$\mathbf{Z}_s = 6 \,||\, (2 + j4) = \frac{6(2 + j4)}{8 + j4} = 2.4 + j1.8$$

$$\mathbf{V}_s = \mathbf{I}_s \mathbf{Z}_s = (6 - j12)(2.4 + j1.8) = 36 - j18 = 18(2 - j)$$

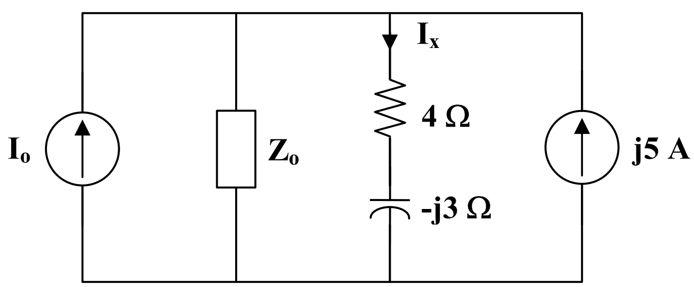
With these, we transform the current source on the left hand side of the circuit to a voltage source. We obtain the circuit in Fig. (b).



(b)

Let
$$\mathbf{Z}_o = \mathbf{Z}_s - j2 = 2.4 - j0.2 = 0.2(12 - j)$$
$$\mathbf{I}_o = \frac{\mathbf{V}_s}{\mathbf{Z}_o} = \frac{18(2-j)}{0.2(12-j)} = 15.517 - j6.207$$

With these, we transform the voltage source in Fig. (b) to a current source. We obtain the circuit in Fig. (c).



(c)

Using current division,
$$\mathbf{I}_x = \frac{\mathbf{Z}_o}{\mathbf{Z}_o + 4 - j3}(\mathbf{I}_o + j5) = \frac{2.4 - j0.2}{6.4 - j3.2}(15.517 - j1.207)$$
$$\mathbf{I}_x = 5 + j1.5625 = \underline{\mathbf{5.238\angle 17.35^\circ\ A}}$$

Chapter 10, Problem 53.

Use the concept of source transformation to find $\mathbf{V}_o$ in the circuit of Fig. 10.97.

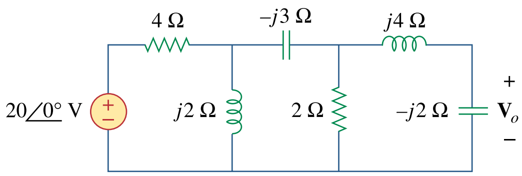
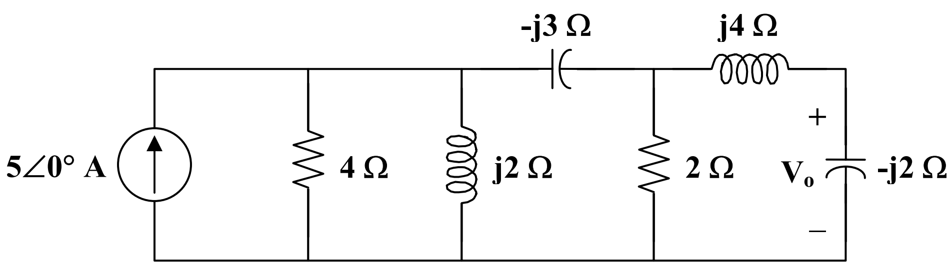


Figure 10.97
For Prob. 10.53.

Chapter 10, Solution 53.

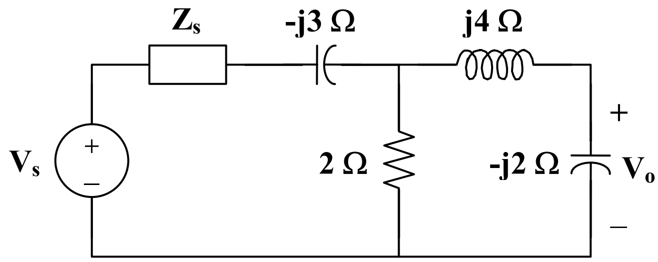
We transform the voltage source to a current source to obtain the circuit in Fig. (a).



(a)

Let
$$\mathbf{Z}_s = 4 \,||\, j2 = \frac{j8}{4 + j2} = 0.8 + j1.6$$
$$\mathbf{V}_s = (5\angle 0°)\,\mathbf{Z}_s = (5)(0.8 + j1.6) = 4 + j8$$

With these, the current source is transformed so that the circuit becomes that shown in Fig. (b).

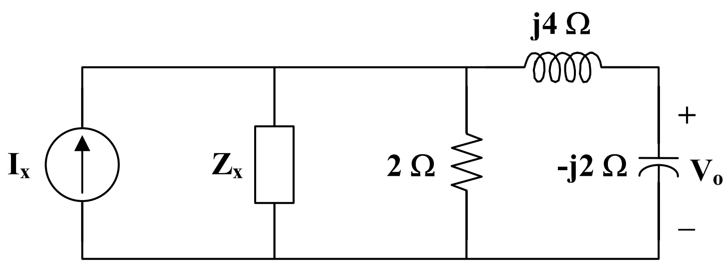


(b)

Let $\mathbf{Z}_x = \mathbf{Z}_s - j3 = 0.8 - j1.4$

$$\mathbf{I}_x = \frac{\mathbf{V}_s}{\mathbf{Z}_s} = \frac{4 + j8}{0.8 - j1.4} = -3.0769 + j4.6154$$

With these, we transform the voltage source in Fig. (b) to obtain the circuit in Fig. (c).

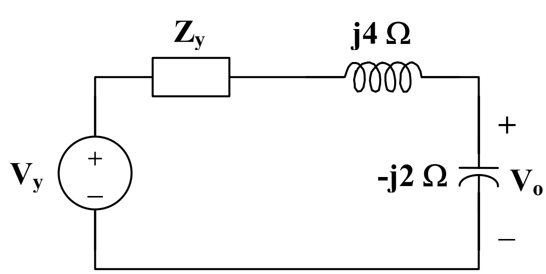


(c)

Let $\mathbf{Z}_y = 2 \parallel \mathbf{Z}_x = \dfrac{1.6 - j2.8}{2.8 - j1.4} = 0.8571 - j0.5714$

$$\mathbf{V}_y = \mathbf{I}_x \, \mathbf{Z}_y = (-3.0769 + j4.6154) \cdot (0.8571 - j0.5714) = j5.7143$$

With these, we transform the current source to obtain the circuit in Fig. (d).
Using current division,



(d)

$$\mathbf{V}_o = \frac{-j2}{\mathbf{Z}_y + j4 - j2}\mathbf{V}_y = \frac{-j2\,(j5.7143)}{0.8571 - j0.5714 + j4 - j2} = \underline{\mathbf{(3.529 - j5.883)\ V}}$$

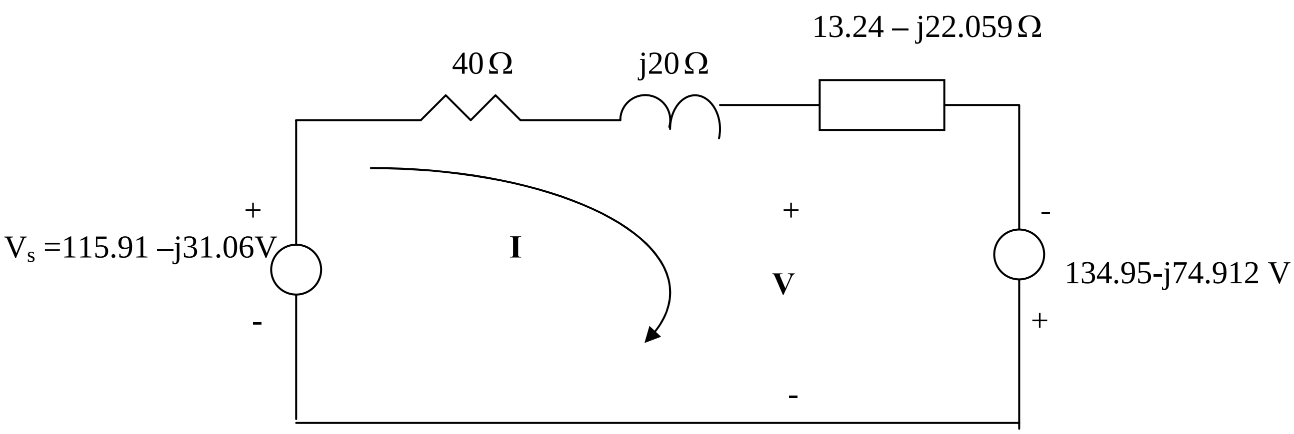
Chapter 10, Problem 54.

Rework Prob. 10.7 using source transformation.

Chapter 10, Solution 54.

$$50//(-j30) = \frac{50x(-j30)}{50-j30} = 13.24 - j22.059$$

We convert the current source to voltage source and obtain the circuit below.



Applying KVL gives

-115.91 + j31.058 + (53.24-j2.059)I -134.95 + j74.912 = 0

or $\quad I = \dfrac{-250.86 + j105.97}{53.24 - j2.059} = -4.7817 + j1.8055$

But $\; -V_s + (40 + j20)I + V = 0 \quad \longrightarrow \quad V = V_s - (40 + j20)I$

$$V = 115.91 - j31.05 - (40 + j20)(-4.7817 + j1.8055) = \underline{124.06\angle -154^o \text{ V}}$$

which agrees with the result in Prob. 10.7.

Chapter 10, Problem 55.
Find the Thevenin and Norton equivalent circuits at terminals *a-b* for each of the circuits in Fig. 10.98.

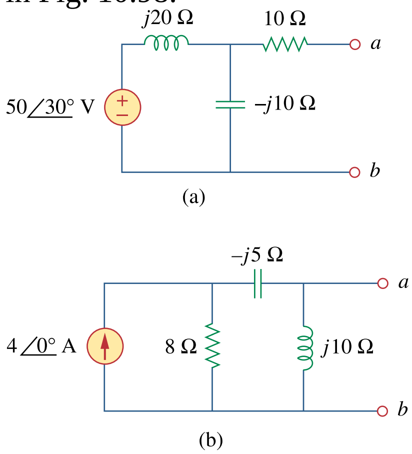
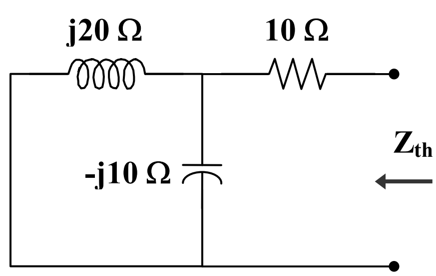


Figure 10.98
For Prob. 10.55.

Chapter 10, Solution 55.

(a) To find $\mathbf{Z}_{th}$, consider the circuit in Fig. (a).

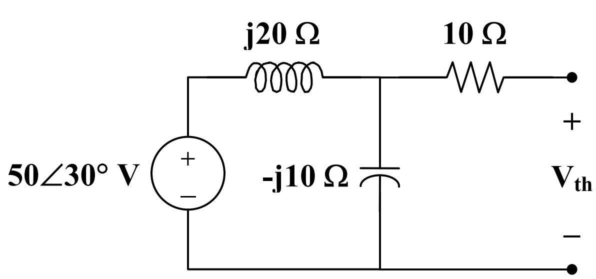


(a)

$$\mathbf{Z}_N = \mathbf{Z}_{th} = 10 + j20 \,\|\, (-j10) = 10 + \frac{(j20)(-j10)}{j20 - j10}$$

$$= 10 - j20 = \underline{\mathbf{22.36\angle{-63.43^\circ}\ \Omega}}$$

To find $\mathbf{V}_{th}$, consider the circuit in Fig. (b).

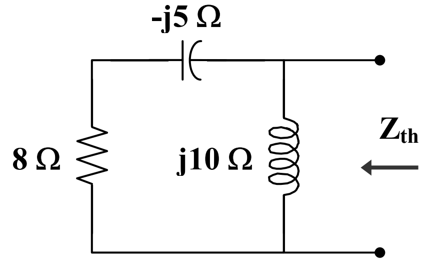


(b)

$$\mathbf{V}_{th} = \frac{-j10}{j20 - j10}(50\angle 30°) = \underline{\mathbf{-50\angle 30°\ V}}$$

$$\mathbf{I}_N = \frac{\mathbf{V}_{th}}{\mathbf{Z}_{th}} = \frac{-50\angle 30°}{22.36\angle -63.43°} = \underline{\mathbf{2.236\angle 273.4°\ A}}$$

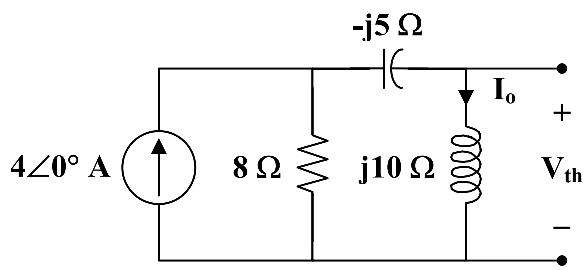
(b) To find $\mathbf{Z}_{th}$, consider the circuit in Fig. (c).



(c)

$$\mathbf{Z}_N = \mathbf{Z}_{th} = j10 \,\|\, (8 - j5) = \frac{(j10)(8 - j5)}{j10 + 8 - j5} = \underline{\mathbf{10\angle 26°\ \Omega}}$$

To obtain $\mathbf{V}_{th}$, consider the circuit in Fig. (d).



(d)

By current division,

$$\mathbf{I}_o = \frac{8}{8 + j10 - j5}(4\angle 0°) = \frac{32}{8 + j5}$$

$$\mathbf{V}_{th} = j10\,\mathbf{I}_o = \frac{j320}{8 + j5} = \underline{\mathbf{33.92\angle 58°\ V}}$$

$$\mathbf{I}_N = \frac{\mathbf{V}_{th}}{\mathbf{Z}_{th}} = \frac{33.92\angle 58°}{10\angle 26°} = \underline{\mathbf{3.392\angle 32°\ A}}$$

Chapter 10, Problem 56.
For each of the circuits in Fig. 10.99, obtain Thevenin and Norton equivalent circuits at terminals *a-b*.

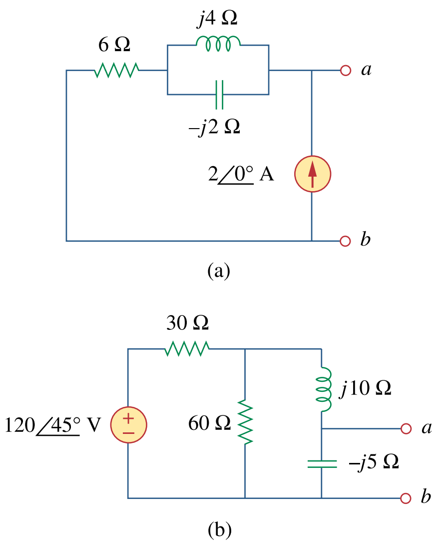
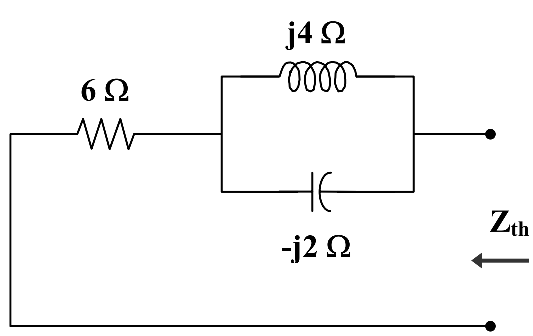


Figure 10.99
For Prob. 10.56.

Chapter 10, Solution 56.
(a) To find $\mathbf{Z}_{th}$, consider the circuit in Fig. (a).



(a)

$$\mathbf{Z}_N = \mathbf{Z}_{th} = 6 + j4 \,||\, (-j2) = 6 + \frac{(j4)(-j2)}{j4 - j2} = 6 - j4$$

$$= \underline{\mathbf{7.211\angle{-33.69}° \ \Omega}}$$

By placing short circuit at terminals a-b, we obtain,
$\mathbf{I}_N = \underline{\mathbf{2\angle 0° \ A}}$

$$\mathbf{V}_{th} = \mathbf{Z}_{th}\,\mathbf{I}_{th} = (7.211\angle{-33.69}°)(2\angle 0°) = \underline{\mathbf{14.422\angle{-33.69}° \ V}}$$

Chapter 10, Problem 57.

Find the Thevenin and Norton equivalent circuits for the circuit shown in Fig. 10.100.

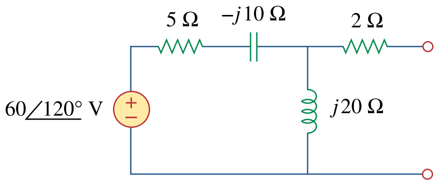
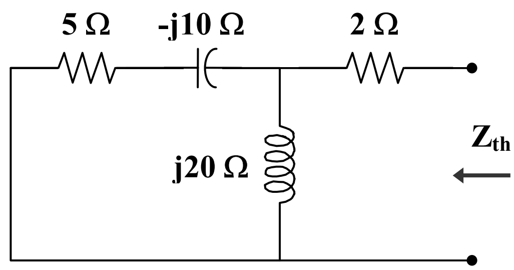


Figure 10.100
For Prob. 10.57.

Chapter 10, Solution 57.

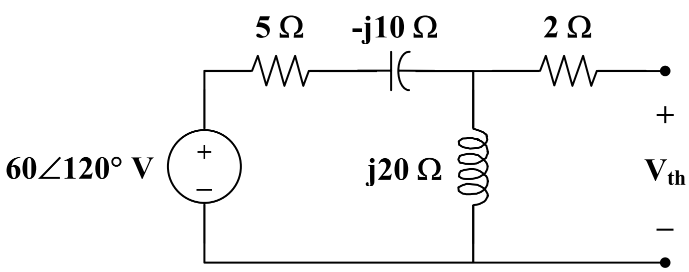
To find $\mathbf{Z}_{th}$, consider the circuit in Fig. (a).



(a)

$$\mathbf{Z}_N = \mathbf{Z}_{th} = 2 + j20 \,||\, (5 - j10) = 2 + \frac{(j20)(5 - j10)}{5 + j10}$$
$$= 18 - j12 = \underline{\mathbf{21.63\angle\text{-}33.7^\circ\ \Omega}}$$

To find $\mathbf{V}_{th}$, consider the circuit in Fig. (b).



(b)

$$\mathbf{V}_{th} = \frac{j20}{5 - j10 + j20}(60\angle 120^\circ) = \frac{j4}{1 + j2}(60\angle 120^\circ)$$
$$= \underline{\mathbf{107.3\angle 146.56^\circ\ V}}$$

$$\mathbf{I}_N = \frac{\mathbf{V}_{th}}{\mathbf{Z}_{th}} = \frac{107.3\angle 146.56^\circ}{21.633\angle -33.7^\circ} = \underline{\mathbf{4.961\angle\text{-}179.7^\circ\ A}}$$

Chapter 10, Problem 58.
For the circuit depicted in Fig. 10.101, find the Thevenin equivalent circuit at terminals *a*-*b*.

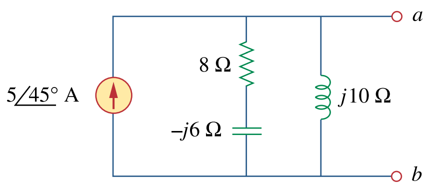
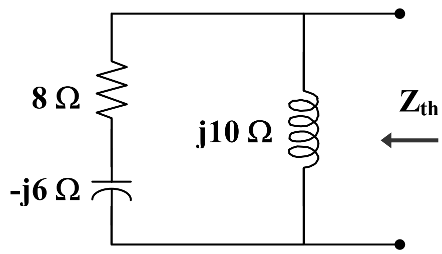


Figure 10.101
For Prob. 10.58.

Chapter 10, Solution 58.

Consider the circuit in Fig. (a) to find $\mathbf{Z}_{th}$.

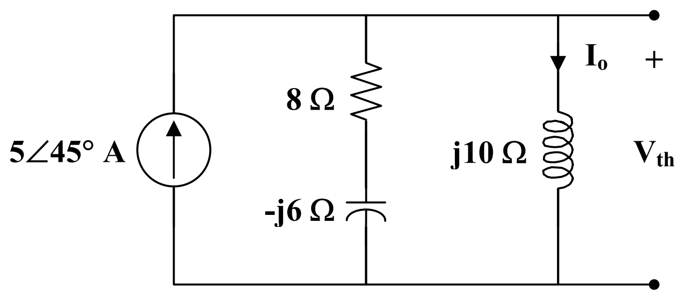


(a)

$$\mathbf{Z}_{th} = j10 \parallel (8 - j6) = \frac{(j10)(8 - j6)}{8 + j4} = 5(2 + j)$$

$$= \underline{\mathbf{11.18\angle 26.56^\circ\ \Omega}}$$

Consider the circuit in Fig. (b) to find $\mathbf{V}_{th}$.



(b)

$$\mathbf{I}_o = \frac{8 - j6}{8 - j6 + j10}(5\angle 45^\circ) = \frac{4 - j3}{4 + j2}(5\angle 45^\circ)$$

$$\mathbf{V}_{th} = j10\mathbf{I}_o = \frac{(j10)(4 - j3)(5\angle 45^\circ)}{(2)(2 + j)} = \underline{\mathbf{55.9\angle 71.56^\circ\ V}}$$

Chapter 10, Problem 59.

Calculate the output impedance of the circuit shown in Fig. 10.102.

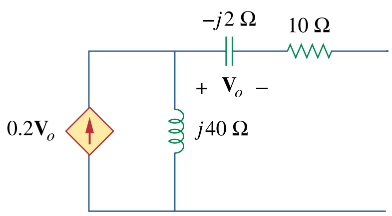
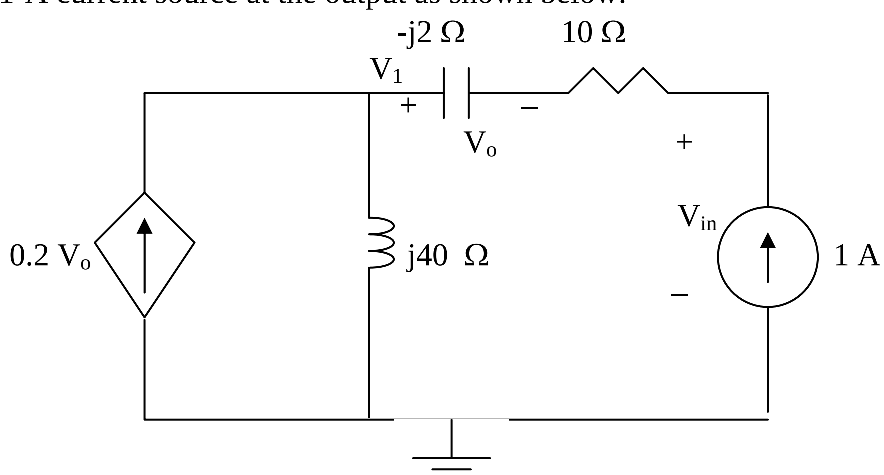


Figure 10.102
For Prob. 10.59.

Chapter 10, Solution 59.

Insert a 1-A current source at the output as shown below.



$$0.2v_o + 1 = \frac{V_1}{j40}$$

But $v_o = -1(-j2) = j2$

$$j2x0.2 + 1 = \frac{V_1}{j40} \longrightarrow V_1 = -16 + j40$$

$V_{in} = V_1 - V_o + 10 = -6 + j38 = 1xZ_{in}$

$Z_{in} =$ **$\underline{-6 + j38\ \Omega}$**.

Chapter 10, Problem 60.

Find the Thevenin equivalent of the circuit in Fig. 10.103 as seen from:
(a) terminals *a-b*   (b) terminals *c-d*

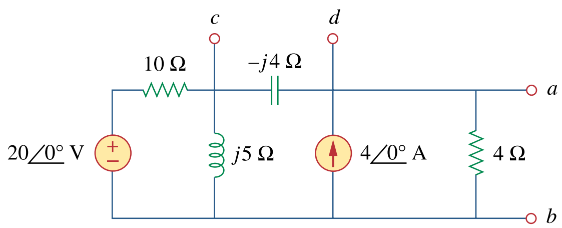
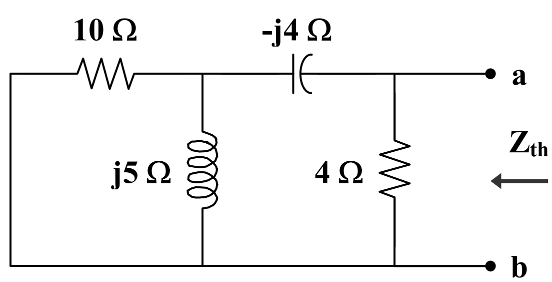


Figure 10.103
For Prob. 10.60.

Chapter 10, Solution 60.

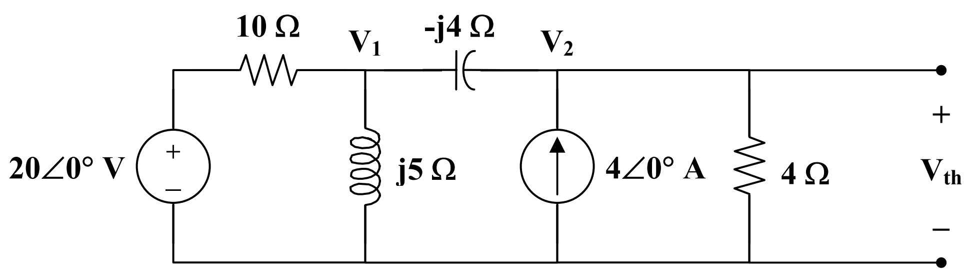
(a) To find $\mathbf{Z}_{th}$, consider the circuit in Fig. (a).



(a)

$$\mathbf{Z}_{th} = 4 \,||\, (-j4 + 10 \,||\, j5) = 4 \,||\, (-j4 + 2 + j4)$$
$$\mathbf{Z}_{th} = 4 \,||\, 2 = \underline{\mathbf{1.333\ \Omega}}$$

To find $\mathbf{V}_{th}$, consider the circuit in Fig. (b).



(b)

At node 1,

$$\frac{20-\mathbf{V}_1}{10} = \frac{\mathbf{V}_1}{\text{j}5} + \frac{\mathbf{V}_1-\mathbf{V}_2}{\text{- j}4}$$

$$(1+\text{j}0.5)\,\mathbf{V}_1 - \text{j}2.5\,\mathbf{V}_2 = 20 \qquad (1)$$

At node 2,

$$4 + \frac{\mathbf{V}_1-\mathbf{V}_2}{\text{- j}4} = \frac{\mathbf{V}_2}{4}$$

$$\mathbf{V}_1 = (1-\text{j})\,\mathbf{V}_2 + \text{j}16 \qquad (2)$$

Substituting (2) into (1) leads to

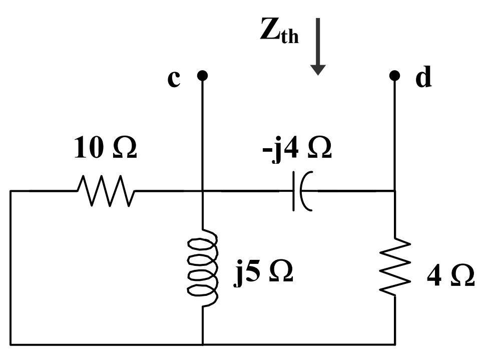
$$28 - \text{j}16 = (1.5-\text{j}3)\,\mathbf{V}_2$$

$$\mathbf{V}_2 = \frac{28-\text{j}16}{1.5-\text{j}3} = 8 + \text{j}5.333$$

Therefore,

$$\mathbf{V}_{th} = \mathbf{V}_2 = \underline{\mathbf{9.615\angle 33.69° \ V}}$$

(b) To find $\mathbf{Z}_{th}$, consider the circuit in Fig. (c).



(c)

$$\mathbf{Z}_{th} = \text{-j}4 \,||\, (4 + 10 \,||\, \text{j}5) = \text{-j}4 \,||\left(4 + \frac{\text{j}10}{2+\text{j}}\right)$$

$$\mathbf{Z}_{th} = \text{-j}4 \,||\, (6+\text{j}4) = \frac{\text{- j}4}{6}(6+\text{j}4) = \underline{\mathbf{2.667 - j4\ \Omega}}$$

To find $\mathbf{V}_{th}$, we will make use of the result in part (a).

$$\mathbf{V}_2 = 8 + \text{j}5.333 = (8/3)\,(3+\text{j}2)$$

$$\mathbf{V}_1 = (1-\text{j})\,\mathbf{V}_2 + \text{j}16 = \text{j}16 + (8/3)(5-\text{j})$$

$$\mathbf{V}_{th} = \mathbf{V}_1 - \mathbf{V}_2 = 16/3 + \text{j}8 = \underline{\mathbf{9.614\angle 56.31° \ V}}$$

Chapter 10, Problem 61.

Find the Thevenin equivalent at terminals *a*-*b* of the circuit in Fig. 10.104.

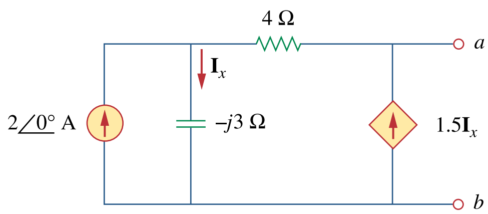
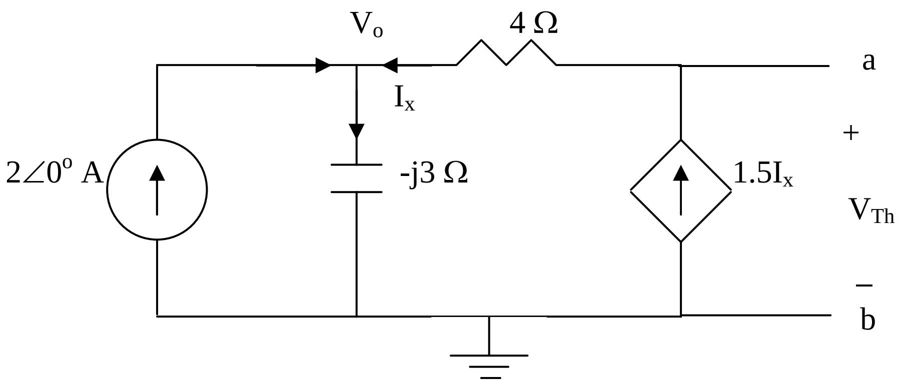


Figure 10.104
For Prob. 10.61.

Chapter 10, Solution 61.

To find V_{Th}, consider the circuit below

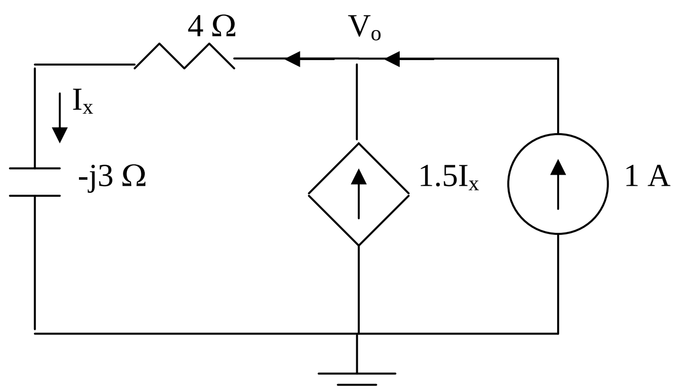


$2 + 1.5I_x = I_x \longrightarrow I_x = -4$

But $V_o = -j3I_x = j12$

$V_{Th} = V_o + 6I_x = \underline{j12 - 24 \text{ V}}$

To find Z_{Th}, consider the circuit shown below.



$1+1.5\,I_x = I_x \Rightarrow I_x = -2$

$-V_o + I_x(4 - j3) = 0 \longrightarrow V_o = -8 + j6$

$$Z_{Th} = \frac{V_o}{1} = \underline{-8 + j6\ \Omega}$$

Chapter 10, Problem 62.

Using Thevenin's theorem, find v_o in the circuit of Fig. 10.105.

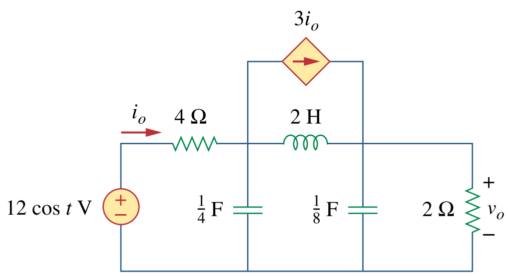


Figure 10.105
For Prob. 10.62.

Chapter 10, Solution 62.

First, we transform the circuit to the frequency domain.

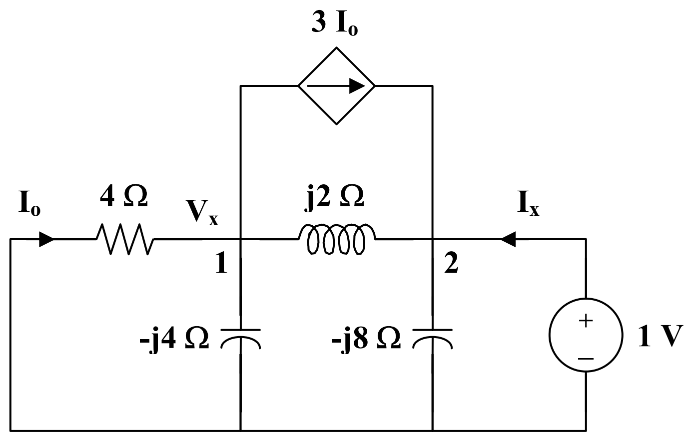
$$12\cos(t) \longrightarrow 12\angle 0°, \quad \omega = 1$$

$$2\text{ H} \longrightarrow j\omega L = j2$$

$$\frac{1}{4}\text{ F} \longrightarrow \frac{1}{j\omega C} = \text{-}j4$$

$$\frac{1}{8}\text{ F} \longrightarrow \frac{1}{j\omega C} = \text{-}j8$$

To find $\mathbf{Z}_{th}$, consider the circuit in Fig. (a).



(a)

At node 1,

$$\frac{\mathbf{V}_x}{4} + \frac{\mathbf{V}_x}{\text{-}j4} + 3\mathbf{I}_o = \frac{1 - \mathbf{V}_x}{j2}, \qquad \text{where} \quad \mathbf{I}_o = \frac{\text{-}\mathbf{V}_x}{4}$$

Thus,

$$\frac{\mathbf{V}_x}{\text{-}j4} - \frac{2\mathbf{V}_x}{4} = \frac{1 - \mathbf{V}_x}{j2}$$

$$\mathbf{V}_x = 0.4 + j0.8$$

At node 2,

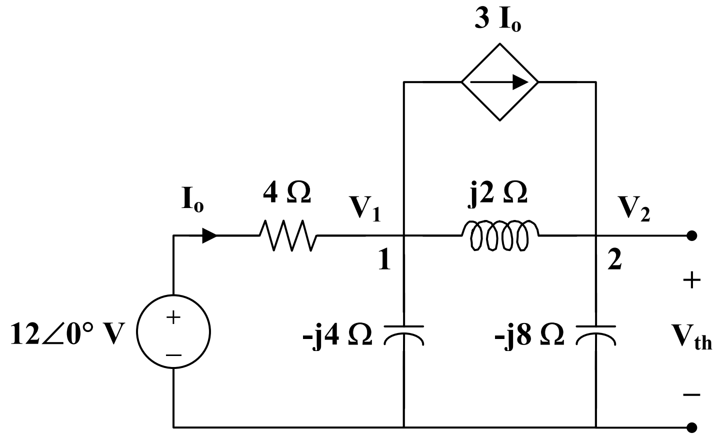
$$\mathbf{I}_x + 3\mathbf{I}_o = \frac{1}{\text{-}j8} + \frac{1 - \mathbf{V}_x}{j2}$$

$$\mathbf{I}_x = (0.75 + j0.5)\,\mathbf{V}_x - j\frac{3}{8}$$

$$\mathbf{I}_x = \text{-}0.1 + j0.425$$

$$\mathbf{Z}_{th} = \frac{1}{\mathbf{I}_x} = \text{-}0.5246 - j2.229 = 2.29\angle\text{-}103.24°\ \Omega$$

To find $\mathbf{V}_{th}$, consider the circuit in Fig. (b).



(b)

At node 1,

$$\frac{12-\mathbf{V}_1}{4} = 3\mathbf{I}_o + \frac{\mathbf{V}_1}{-j4} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{j2}, \qquad \text{where} \quad \mathbf{I}_o = \frac{12-\mathbf{V}_1}{4}$$

$$24 = (2+j)\,\mathbf{V}_1 - j2\,\mathbf{V}_2 \qquad (1)$$

At node 2,

$$\frac{\mathbf{V}_1 - \mathbf{V}_2}{j2} + 3\mathbf{I}_o = \frac{\mathbf{V}_2}{-j8}$$

$$72 = (6+j4)\,\mathbf{V}_1 - j3\,\mathbf{V}_2 \qquad (2)$$

From (1) and (2),

$$\begin{bmatrix} 24 \\ 72 \end{bmatrix} = \begin{bmatrix} 2+j & -j2 \\ 6+j4 & -j3 \end{bmatrix} \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix}$$

$$\Delta = -5 + j6, \qquad \Delta_2 = -j24$$

$$\mathbf{V}_{th} = \mathbf{V}_2 = \frac{\Delta_2}{\Delta} = 3.073\angle -219.8°$$

Thus,

$$\mathbf{V}_o = \frac{2}{2+\mathbf{Z}_{th}}\mathbf{V}_{th} = \frac{(2)(3.073\angle -219.8°)}{1.4754 - j2.229}$$

$$\mathbf{V}_o = \frac{6.146\angle -219.8°}{2.673\angle -56.5°} = 2.3\angle -163.3°$$

Therefore, $\quad v_o =$ **<u>2.3 cos(t – 163.3°) V</u>**

Chapter 10, Solution 63.

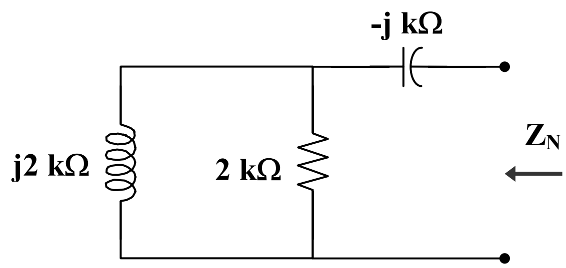
Transform the circuit to the frequency domain.

$$4\cos(200t + 30°) \longrightarrow 4\angle 30°, \quad \omega = 200$$

$$10\text{ H} \longrightarrow j\omega L = j(200)(10) = j2\text{ k}\Omega$$

$$5\ \mu\text{F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(200)(5\times 10^{-6})} = -j\text{ k}\Omega$$

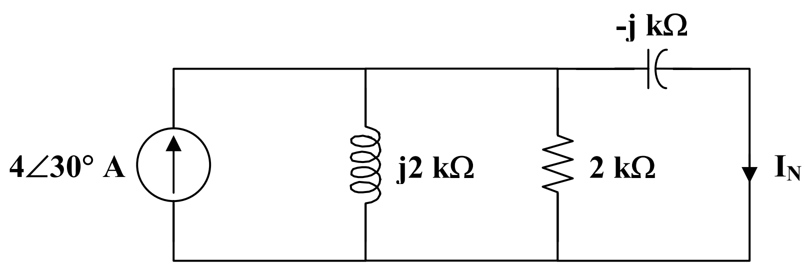
$\mathbf{Z}_N$ is found using the circuit in Fig. (a).



(a)

$$\mathbf{Z}_N = -j + 2 \,\|\, j2 = -j + 1 + j = 1\text{ k}\Omega$$

We find $\mathbf{I}_N$ using the circuit in Fig. (b).



(b)

$$j2 \,\|\, 2 = 1 + j$$

By the current division principle,

$$\mathbf{I}_N = \frac{1+j}{1+j-j}(4\angle 30°) = 5.657\angle 75°$$

Therefore,

i_N = **<u>5.657 cos(200t + 75°) A</u>**

$\mathbf{Z}_N$ = **<u>1 kΩ</u>**

Chapter 10, Problem 64.

For the circuit shown in Fig. 10.107, find the Norton equivalent circuit at terminals *a-b*.

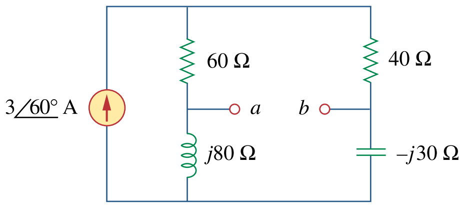
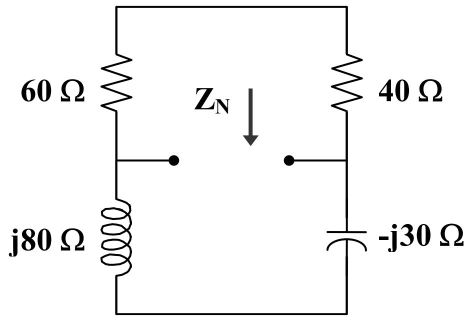


Figure 10.107
For Prob. 10.64.

Chapter 10, Solution 64.

$\mathbf{Z}_N$ is obtained from the circuit in Fig. (a).

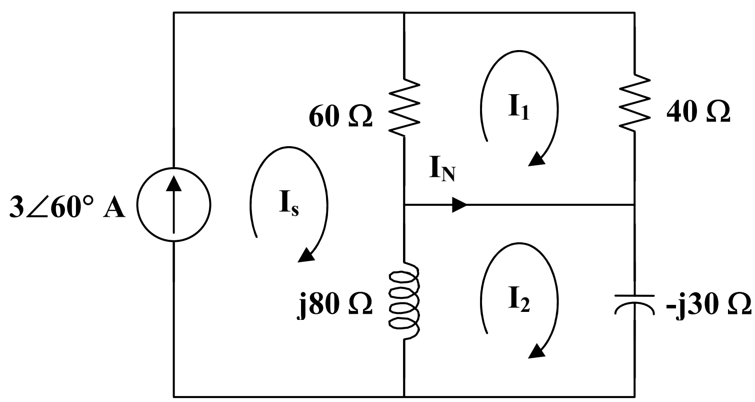


(a)

$$\mathbf{Z}_N = (60 + 40) \,||\, (j80 - j30) = 100 \,||\, j50 = \frac{(100)(j50)}{100 + j50}$$

$$\mathbf{Z}_N = 20 + j40 = \underline{\mathbf{44.72\angle 63.43°\ \Omega}}$$

To find $\mathbf{I}_N$, consider the circuit in Fig. (b).



(b)

$$\mathbf{I}_s = 3\angle 60°$$

For mesh 1,

$$100\mathbf{I}_1 - 60\mathbf{I}_s = 0$$

$$\mathbf{I}_1 = 1.8\angle 60°$$

For mesh 2,

$$(j80 - j30)\mathbf{I}_2 - j80\mathbf{I}_s = 0$$

$$\mathbf{I}_2 = 4.8\angle 60°$$

$I_N = I_2 – I_1 = \underline{\mathbf{3\angle 60°\ A}}$

Chapter 10, Problem 65.

Compute i_o in Fig. 10.108 using Norton's theorem.

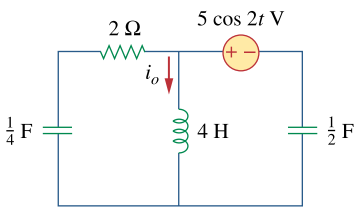


Figure 10.108
For Prob. 10.65.

Chapter 10, Solution 65.

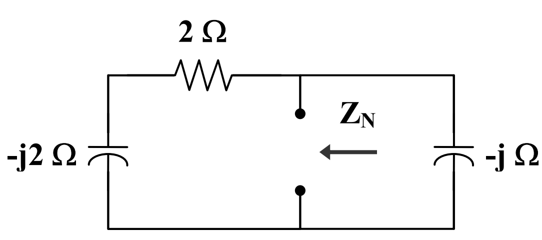
$$5\cos(2t) \longrightarrow 5\angle 0°, \quad \omega = 2$$

$$4\text{ H} \longrightarrow j\omega L = j(2)(4) = j8$$

$$\frac{1}{4}\text{ F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(2)(1/4)} = -j2$$

$$\frac{1}{2}\text{ F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(2)(1/2)} = -j$$

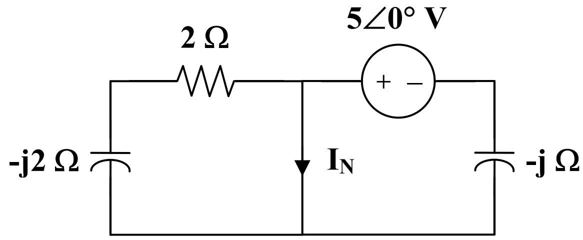
To find $\mathbf{Z}_N$, consider the circuit in Fig. (a).



(a)

$$\mathbf{Z}_N = -j \,\|\, (2 - j2) = \frac{-j(2 - j2)}{2 - j3} = \frac{1}{13}(2 - j10)$$

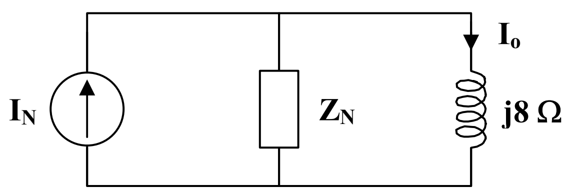
To find $\mathbf{I}_N$, consider the circuit in Fig. (b).



(b)

$$\mathbf{I}_N = \frac{5\angle 0^\circ}{-j} = j5$$

The Norton equivalent of the circuit is shown in Fig. (c).



(c)

Using current division,

$$\mathbf{I}_o = \frac{\mathbf{Z}_N}{\mathbf{Z}_N + j8}\mathbf{I}_N = \frac{(1/13)(2 - j10)(j5)}{(1/13)(2 - j10) + j8} = \frac{50 + j10}{2 + j94}$$

$$\mathbf{I}_o = 0.1176 - j0.5294 = 0542\angle -77.47^\circ$$

Therefore, $i_o =$ <u>**542 cos(2t – 77.47°) mA**</u>

Chapter 10, Problem 66.

At terminals *a-b*, obtain Thevenin and Norton equivalent circuits for the network depicted in Fig. 10.109. Take $\omega = 10$ rad/s.

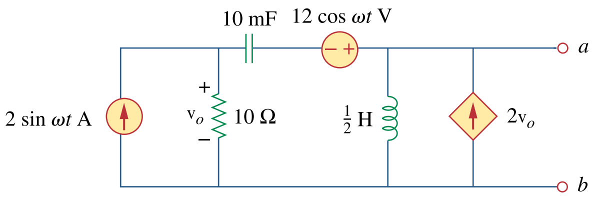


Figure 10.109
For Prob. 10.66.

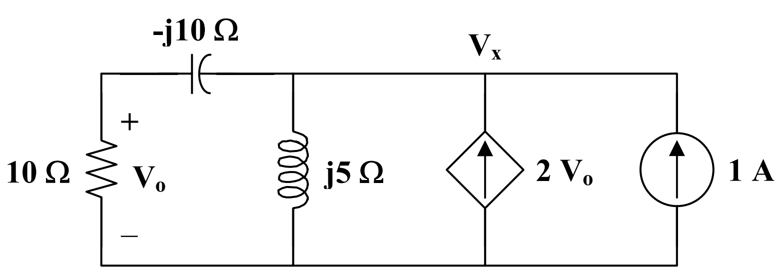
Chapter 10, Solution 66.

$$\omega = 10$$

$$0.5\text{ H} \longrightarrow j\omega L = j(10)(0.5) = j5$$

$$10\text{ mF} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(10)(10\times 10^{-3})} = \text{-}j10$$

To find $\mathbf{Z}_{th}$, consider the circuit in Fig. (a).



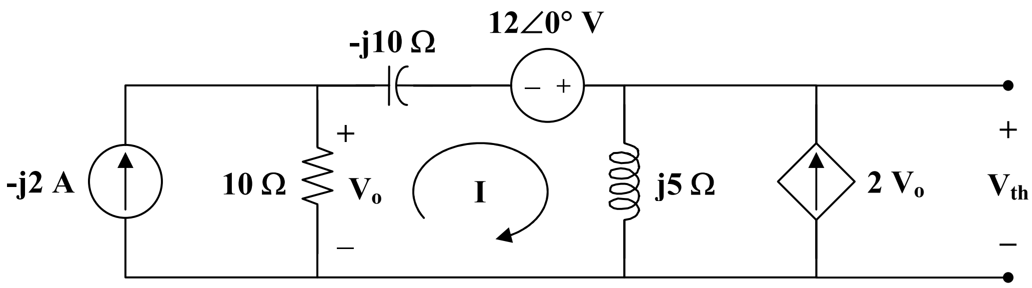
(a)

$$1 + 2\mathbf{V}_o = \frac{\mathbf{V}_x}{j5} + \frac{\mathbf{V}_x}{10 - j10}, \qquad \text{where } \mathbf{V}_o = \frac{10\mathbf{V}_x}{10 - j10}$$

$$1 + \frac{19\mathbf{V}_x}{10 - j10} = \frac{\mathbf{V}_x}{j5} \longrightarrow \mathbf{V}_x = \frac{\text{-}10 + j10}{21 + j2}$$

$$\mathbf{Z}_N = \mathbf{Z}_{th} = \frac{\mathbf{V}_x}{1} = \frac{14.142\angle 135^\circ}{21.095\angle 5.44^\circ} = \underline{\mathbf{0.67\angle 129.56^\circ\ \Omega}}$$

To find $\mathbf{V}_{th}$ and $\mathbf{I}_N$, consider the circuit in Fig. (b).



(b)

$$(10 - j10 + j5)\mathbf{I} - (10)(\text{-}j2) + j5(2\mathbf{V}_o) - 12 = 0$$

where $$\mathbf{V}_o = (10)(\text{-}j2 - \mathbf{I})$$

Thus,

$$(10 - j105)\mathbf{I} = \text{-}188 - j20$$

$$\mathbf{I} = \frac{188 + j20}{\text{-}10 + j105}$$

$$\mathbf{V}_{th} = j5(\mathbf{I} + 2\mathbf{V}_o) = j5(-19\mathbf{I} - j40) = -j95\mathbf{I} + 200$$

$$\mathbf{V}_{th} = \frac{-j95(188 + j20)}{\text{-}10 + j105} + 200 = 29.73 + j1.8723$$

$$\mathbf{V}_{th} = \underline{\mathbf{29.79\angle 3.6^\circ\ V}}$$

$$\mathbf{I}_N = \frac{\mathbf{V}_{th}}{\mathbf{Z}_{th}} = \frac{29.79\angle 3.6^\circ}{0.67\angle 129.56^\circ} = \underline{\mathbf{44.46\angle{-125.96^\circ}\ A}}$$

Chapter 10, Problem 67.

Find the Thevenin and Norton equivalent circuits at terminals *a*-*b* in the circuit of Fig. 10.110.

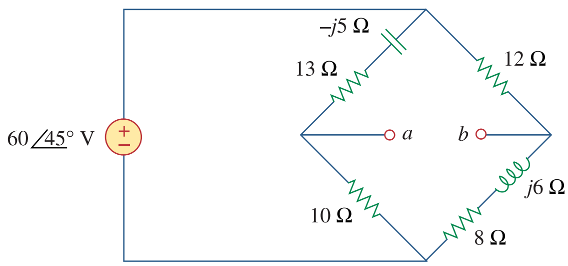


Figure 10.110
For Prob. 10.67.

Chapter 10, Solution 67.

$$Z_N = Z_{Th} = 10//(13-j5) + 12//(8+j6) = \frac{10(13-j5)}{23-j5} + \frac{12(8+j6)}{20+j6} = \underline{11.243 + j1.079\Omega}$$

$$V_a = \frac{10}{23-j5}(60\angle 45^o) = 13.78 + j21.44, \qquad V_b = \frac{(8+j6)}{20+j6}(60\angle 45^o) = 12.069 + j26.08\Omega$$

$$V_{Th} = V_a - V_b = \underline{1.711 - j4.64 = 4.945\angle -69.76^o}\ \text{V},$$

$$I_N = \frac{V_{Th}}{Z_{Th}} = \frac{4.945\angle -69.76°}{11.295\angle 5.48°} = \underline{0.4378\angle -75.24^o}\ \text{A}$$

Chapter 10, Problem 68.

Find the Thevenin equivalent at terminals *a-b* in the circuit of Fig. 10.111.

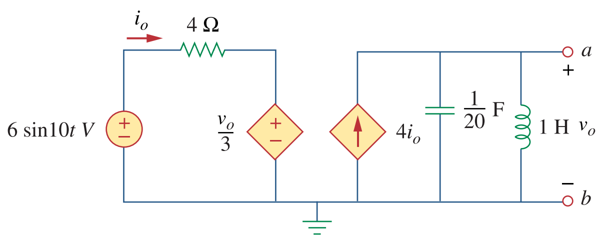


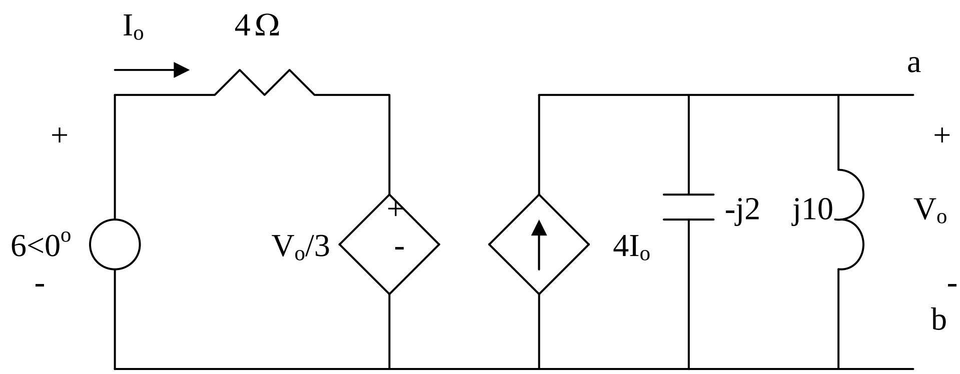
Figure 10.111
For Prob. 10.68.

Chapter 10, Solution 68.

$$1\text{H} \longrightarrow j\omega L = j10x1 = j10$$

$$\frac{1}{20}\text{F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j10x\frac{1}{20}} = -j2$$

We obtain V_{Th} using the circuit below.



$$j10 // (-j2) = \frac{j10(-j2)}{j10 - j2} = -j2.5$$

$$V_o = 4I_o x(-j2.5) = -j10I_o \qquad (1)$$

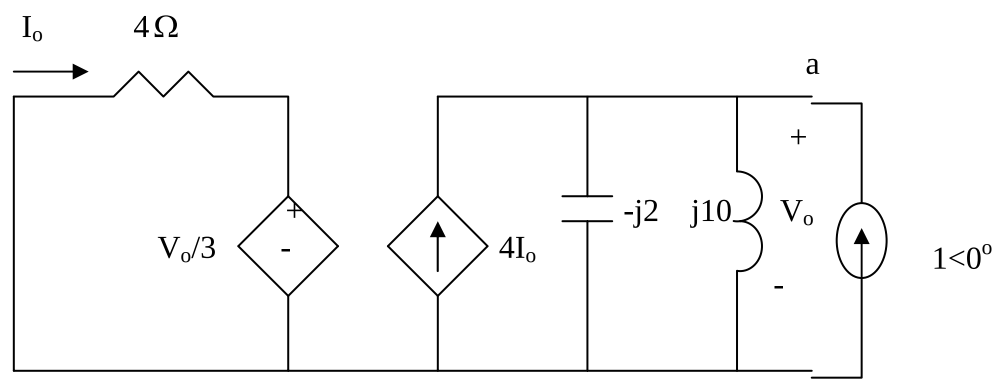
$$-6 + 4I_o + \frac{1}{3}V_o = 0 \qquad (2)$$

Combining (1) and (2) gives

$$I_o = \frac{6}{4 - j10/3}, \qquad V_{Th} = V_o = -j10I_o = \frac{-j60}{4 - j10/3} = 11.52\angle -50.19^o$$

$$\underline{v_{Th} = 11.52\sin(10t - 50.19^o)}$$

To find $R_{Th,}$ we insert a 1-A source at terminals a-b, as shown below.



$$4I_o + \frac{1}{3}V_o = 0 \quad \longrightarrow \quad I_o = -\frac{V_o}{12}$$

$$1 + 4I_o = \frac{V_o}{-j2} + \frac{V_o}{j10}$$

Combining the two equations leads to

$$V_o = \frac{1}{0.333 + j0.4} = 1.2293 - j1.4766$$

$$Z_{Th} = \frac{V_o}{1} = \underline{1.2293 - 1.477\Omega}$$

Chapter 10, Problem 69.

For the differentiator shown in Fig. 10.112, obtain $\mathbf{V}_o/\mathbf{V}_s$. Find $v_o(t)$ when $v_s(t) = \mathbf{V}_m$ sin ωt and $\omega = 1/RC$.

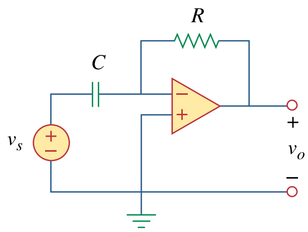


Figure 10.112
For Prob. 10.69.

Chapter 10, Solution 69.

This is an inverting op amp so that

$$\frac{\mathbf{V}_o}{\mathbf{V}_s} = \frac{-\mathbf{Z}_f}{\mathbf{Z}_i} = \frac{-R}{1/j\omega C} = \underline{\mathbf{-j\omega RC}}$$

When $\mathbf{V}_s = V_m$ and $\omega = 1/RC$,

$$\mathbf{V}_o = -j \cdot \frac{1}{RC} \cdot RC \cdot V_m = -jV_m = V_m \angle -90°$$

Therefore,

$$v_o(t) = V_m \sin(\omega t - 90°) = \underline{\mathbf{-V_m \cos(\omega t)}}$$

Chapter 10, Problem 70.

The circuit in Fig. 10.113 is an integrator with a feedback resistor. Calculate $v_o(t)$ if $v_s = 2\cos 4\times 10^4 t$ V.

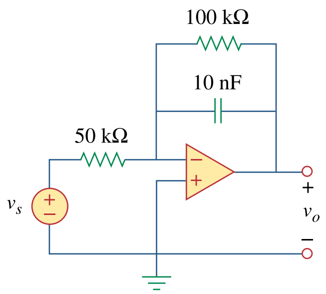


Figure 10.113
For Prob. 10.70.

Chapter 10, Solution 70.

This may also be regarded as an inverting amplifier.

$$2\cos(4\times 10^4 t) \longrightarrow 2\angle 0°, \quad \omega = 4\times 10^4$$

$$10 \text{ nF} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(4\times 10^4)(10\times 10^{-9})} = -j2.5 \text{ k}\Omega$$

$$\frac{\mathbf{V}_o}{\mathbf{V}_s} = \frac{-\mathbf{Z}_f}{\mathbf{Z}_i}$$

where $\mathbf{Z}_i = 50 \text{ k}\Omega$ and $\mathbf{Z}_f = 100\text{k} \parallel (-j2.5\text{k}) = \dfrac{-j100}{40 - j} \text{ k}\Omega$.

Thus, $$\frac{\mathbf{V}_o}{\mathbf{V}_s} = \frac{j2}{40 - j}$$

If $\mathbf{V}_s = 2\angle 0°$,

$$\mathbf{V}_o = \frac{j4}{40 - j} = \frac{4\angle 90°}{40.01\angle -1.43°} = 0.1\angle 91.43°$$

Therefore,

$$v_o(t) = \underline{\mathbf{0.1\cos(4x10^4 t + 91.43°)\ V}}$$

Chapter 10, Problem 71.

Find v_o in the op amp circuit of Fig. 10.114.

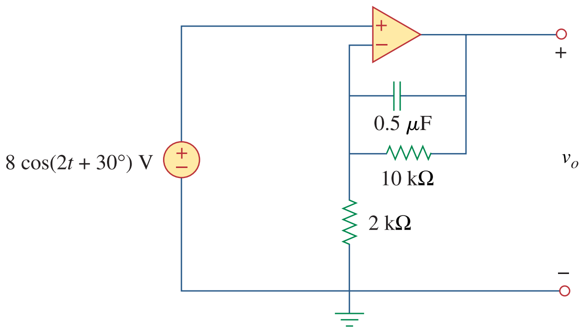


Figure 10.114
For Prob. 10.71.

Chapter 10, Solution 71.

$$8\cos(2t+30^o) \longrightarrow 8\angle 30^o$$

$$0.5\mu F \longrightarrow \frac{1}{j\omega C} = \frac{1}{j2x0.5x10^{-6}} = -j1M\Omega$$

At the inverting terminal,

$$\frac{V_o - 8\angle 30^o}{-j1000k} + \frac{V_o - 8\angle 30^o}{10k} = \frac{8\angle 30^o}{2k} \longrightarrow$$

$$V_o(1-j100) = 8\angle 30 + 800\angle -60° + 4000\angle -60°$$

$$V_o = \frac{6.928 + j4 + 2400 - j4157}{1-j100} = \frac{4800\angle -59.9°}{100\angle -89.43°} = 48\angle 29.53^o$$

$v_o(t)$ = **48cos(2t + 29.53°) V**

Chapter 10, Problem 72.

Compute $i_o(t)$ in the op amp circuit in Fig. 10.115 if $v_s = 4\cos 10^4 t$ V.

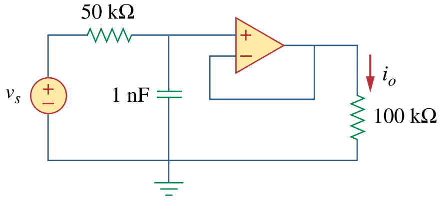


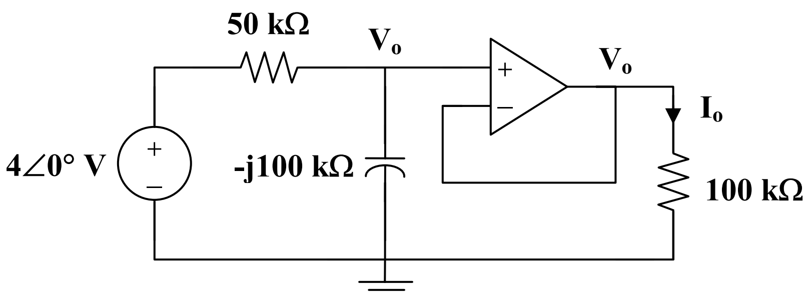
Figure 10.115
For Prob. 10.72.

Chapter 10, Solution 72.

$$4\cos(10^4 t) \longrightarrow 4\angle 0°, \quad \omega = 10^4$$

$$1\text{ nF} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(10^4)(10^{-9})} = \text{-j}100\text{ k}\Omega$$

Consider the circuit as shown below.



At the noninverting node,

$$\frac{4 - \mathbf{V}_o}{50} = \frac{\mathbf{V}_o}{-\text{j}100} \longrightarrow \mathbf{V}_o = \frac{4}{1 + \text{j}0.5}$$

$$\mathbf{I}_o = \frac{\mathbf{V}_o}{100\text{k}} = \frac{4}{(100)(1 + \text{j}0.5)}\text{mA} = 35.78\angle\text{-}26.56°\ \mu\text{A}$$

Therefore,

$$i_o(t) = \underline{\mathbf{35.78\cos(10^4\, t - 26.56°)\ \mu A}}$$

Chapter 10, Problem 73.

If the input impedance is defined as $\mathbf{Z}_{in} = \mathbf{V}_s / \mathbf{I}_s$ find the input impedance of the op amp circuit in Fig. 10.116 when $R_1 = 10\,\text{k}\Omega, R_2 = 20\,\text{k}\Omega, C_1 = 10\,\text{nF}$, and $\omega = 5000\,\text{rad/s}$.

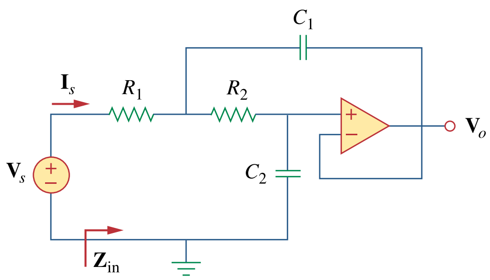


Figure 10.116
For Prob. 10.73.

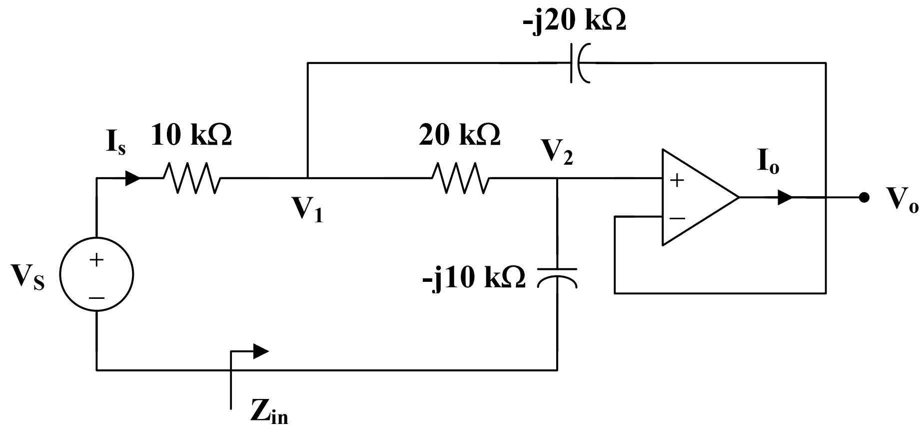
Chapter 10, Solution 73.

As a voltage follower, $\mathbf{V}_2 = \mathbf{V}_o$

$$C_1 = 10\text{ nF} \longrightarrow \frac{1}{j\omega C_1} = \frac{1}{j(5\times 10^3)(10\times 10^{-9})} = -j20\text{ k}\Omega$$

$$C_2 = 20\text{ nF} \longrightarrow \frac{1}{j\omega C_2} = \frac{1}{j(5\times 10^3)(20\times 10^{-9})} = -j10\text{ k}\Omega$$

Consider the circuit in the frequency domain as shown below.



At node 1,

$$\frac{\mathbf{V}_s - \mathbf{V}_1}{10} = \frac{\mathbf{V}_1 - \mathbf{V}_o}{-\,j20} + \frac{\mathbf{V}_1 - \mathbf{V}_o}{20}$$

$$2\,\mathbf{V}_s = (3 + j)\mathbf{V}_1 - (1 + j)\mathbf{V}_o \qquad (1)$$

At node 2,

$$\frac{\mathbf{V}_1 - \mathbf{V}_o}{20} = \frac{\mathbf{V}_o - 0}{-\,j10}$$

$$\mathbf{V}_1 = (1 + j2)\mathbf{V}_o \qquad (2)$$

Substituting (2) into (1) gives

$$2\,\mathbf{V}_s = j6\mathbf{V}_o \quad \text{or} \quad \mathbf{V}_o = -j\frac{1}{3}\mathbf{V}_s$$

$$\mathbf{V}_1 = (1 + j2)\mathbf{V}_o = \left(\frac{2}{3} - j\frac{1}{3}\right)\mathbf{V}_s$$

$$\mathbf{I}_S = \frac{\mathbf{V}_S - \mathbf{V}_1}{10k} = \frac{(1/3)(1 + j)}{10k}\mathbf{V}_S$$

$$\frac{\mathbf{I}_S}{\mathbf{V}_S} = \frac{1 + j}{30k}$$

$$\mathbf{Z}_{in} = \frac{\mathbf{V}_S}{\mathbf{I}_S} = \frac{30k}{1 + j} = 15(1 - j)\,k$$

$$\mathbf{Z}_{in} = \underline{\mathbf{21.21\angle{-45^\circ}\ k\Omega}}$$

Chapter 10, Problem 74.

Evaluate the voltage gain $\mathbf{A}_v = \mathbf{V}_o/\mathbf{V}_s$ in the op amp circuit of Fig. 10.117. Find $\mathbf{A}_v$ at $\omega = 0, \omega \to \infty, \omega = 1/R_1C_1$, and $\omega = 1/R_2C_2$.

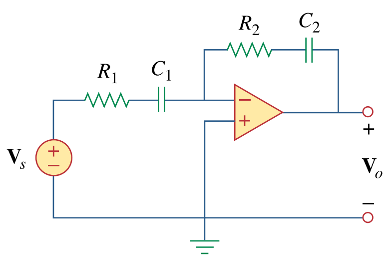


Figure 10.117
For Prob. 10.74.

Chapter 10, Solution 74.

$$\mathbf{Z}_i = R_1 + \frac{1}{j\omega C_1}, \qquad \mathbf{Z}_f = R_2 + \frac{1}{j\omega C_2}$$

$$\mathbf{A}_v = \frac{\mathbf{V}_o}{\mathbf{V}_s} = \frac{-\mathbf{Z}_f}{\mathbf{Z}_i} = -\frac{R_2 + \dfrac{1}{j\omega C_2}}{R_1 + \dfrac{1}{j\omega C_1}} = \underline{-\left(\frac{C_1}{C_2}\right)\left(\frac{1+j\omega R_2C_2}{1+j\omega R_1C_1}\right)}$$

At $\omega = 0$, $\qquad \mathbf{A}_v = \underline{-\dfrac{\mathbf{C_1}}{\mathbf{C_2}}}$

As $\omega \to \infty$, $\qquad \mathbf{A}_v = \underline{-\dfrac{\mathbf{R_2}}{\mathbf{R_1}}}$

At $\omega = \dfrac{1}{R_1C_1}$, $\qquad \mathbf{A}_v = \underline{-\left(\dfrac{\mathbf{C_1}}{\mathbf{C_2}}\right)\left(\dfrac{\mathbf{1 + jR_2C_2/R_1C_1}}{\mathbf{1+j}}\right)}$

At $\omega = \dfrac{1}{R_2C_2}$, $\qquad \mathbf{A}_v = \underline{-\left(\dfrac{\mathbf{C_1}}{\mathbf{C_2}}\right)\left(\dfrac{\mathbf{1+j}}{\mathbf{1 + jR_1C_1/R_2C_2}}\right)}$

Chapter 10, Problem 75.

In the op amp circuit of Fig. 10.118, find the closed-loop gain and phase shift of the output voltage with respect to the input voltage if $C_1 = C_2 = 1\,\text{nF}$, $R_1 = R_2 = 100\,\text{k}\Omega$, $R_3 = 20\ \text{k}\Omega$, $R_4 = 40\ \text{k}\Omega$, and $\omega = 2000$ rad/s.

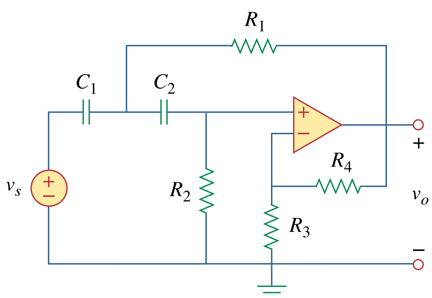


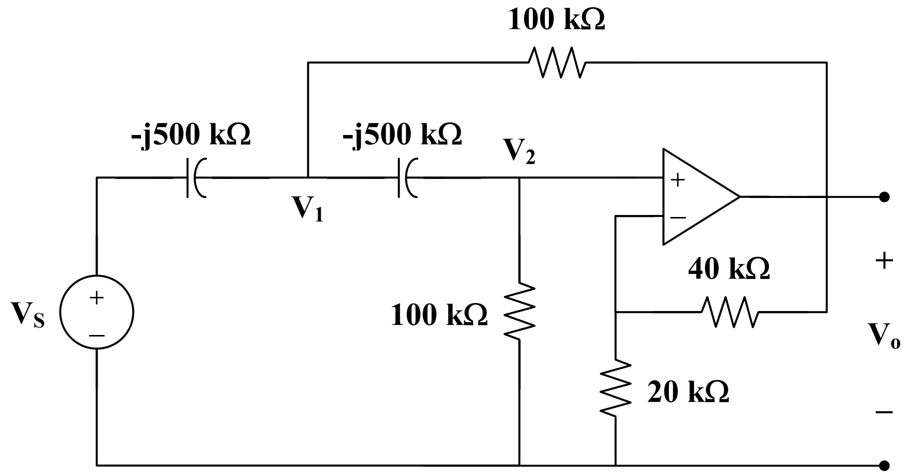
Figure 10.118
For Prob. 10.75.

Chapter 10, Solution 75.

$\omega = 2\times 10^3$

$C_1 = C_2 = 1\,\text{nF} \longrightarrow \dfrac{1}{j\omega C_1} = \dfrac{1}{j(2\times 10^3)(1\times 10^{-9})} = \text{-j500 k}\Omega$

Consider the circuit shown below.



Let $\mathbf{V}_s$ = 10V.

At node 1,

$[(\mathbf{V}_1 - 10)/(-j500k)] + [(\mathbf{V}_1 - \mathbf{V}_o)/10^5] + [(\mathbf{V}_1 - \mathbf{V}_2)/(-j500k)] = 0$

or $(1+j0.4)\mathbf{V}_1 - j0.2\mathbf{V}_2 - \mathbf{V}_o = j2$ (1)

At node 2,

$[(\mathbf{V}_2 - \mathbf{V}_1)/(-j5)] + (\mathbf{V}_2 - 0) = 0$

or $-j0.2\mathbf{V}_1 + (1+j0.2)\mathbf{V}_2 = 0$ or $\mathbf{V}_1 = (1-j5)\mathbf{V}_2$ (2)

But

$$\mathbf{V}_2 = \frac{R_3}{R_3 + R_4}\mathbf{V}_o = \frac{\mathbf{V}_o}{3} \qquad (3)$$

From (2) and (3),

$\mathbf{V}_1 = (0.3333 - j1.6667)\mathbf{V}_o$ (4)

Substituting (3) and (4) into (1),

$(1+j0.4)(0.3333–j1.6667)\mathbf{V}_o – j0.06667\mathbf{V}_o – \mathbf{V}_o = j2$

$(1.077\angle 21.8^\circ)(1.6997\angle –78.69^\circ) = 1.8306\angle –56.89^\circ = 1 – j1.5334$

Thus,

$(1–j1.5334)\mathbf{V}_o – j0.06667\mathbf{V}_o – \mathbf{V}_o = j2$

and, $V_o = j2/(–j1.6601) = –1.2499 = 1.2499\angle 180^\circ$ V

Since $\mathbf{V}_s = 10$,

$$\mathbf{V}_o/\mathbf{V}_s = \underline{\mathbf{0.12499\angle 180^\circ}}.$$

Checking with MATLAB.

```
>> Y=[1+0.4i,-0.2i,-1;1,-1+5i,0;0,-3,1]

Y =

   1.0000 + 0.4000i        0 - 0.2000i  -1.0000
   1.0000            -1.0000 + 5.0000i        0
        0            -3.0000              1.0000

>> I=[2i;0;0]

I =

        0 + 2.0000i
        0
        0

>> V=inv(Y)*I

V =

  -0.4167 + 2.0833i
  -0.4167
  -1.2500 + 0.0000i (this last term is vo)
```

and, the answer checks.

Chapter 10, Problem 76.

Determine $\mathbf{V}_o$ and $\mathbf{I}_o$ in the op amp circuit of Fig. 10.119.

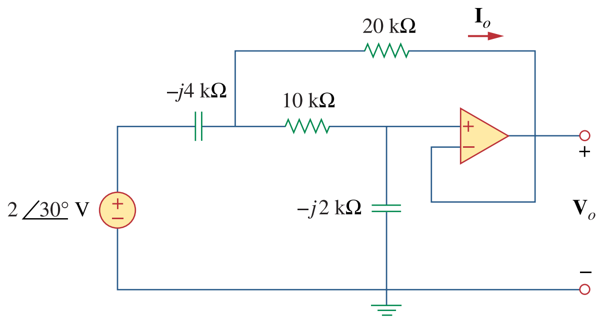


Figure 10.119
For Prob. 10.76.

Chapter 10, Solution 76.

Let the voltage between the -jkΩ capacitor and the 10kΩ resistor be $\mathbf{V}_1$.

$$\frac{2\angle 30^o - V_1}{-j4k} = \frac{V_1 - V_o}{10k} + \frac{V_1 - V_o}{20k} \longrightarrow \quad (1)$$

$$2\angle 30^o = (1 - j0.6)V_1 + j0.6V_o$$
$$= 1.7321 + j1$$

Also,

$$\frac{V_1 - V_o}{10k} = \frac{V_o}{-j2k} \longrightarrow \quad V_1 = (1 + j5)V_o \quad (2)$$

Solving (2) into (1) yields

$$2\angle 30° = (1 - j0.6)(1 + j5)V_o + j0.6V_o = (1 + 3 - j0.6 + j5 + j6)V_o$$
$$= (4+j5)V_o$$
$$V_o = \frac{2\angle 30°}{6.403\angle 51.34°} = \underline{0.3124\angle -21.34^o \text{ V}}$$

```
>> Y=[1-0.6i,0.6i;1,-1-0.5i]

Y =

   1.0000 - 0.6000i        0 + 0.6000i
   1.0000             -1.0000 - 5.0000i

>> I=[1.7321+1i;0]

I =

   1.7321 + 1.0000i
        0

>> V=inv(Y)*I

V =

   0.8593 + 1.3410i
   0.2909 - 0.1137i
```

$= V_o = 0.3123\angle{-21.35}^\circ$V. Answer checks.

Chapter 10, Problem 77.

Compute the closed-loop gain $\mathbf{V}_o/\mathbf{V}_s$ for the op amp circuit of Fig. 10.120.

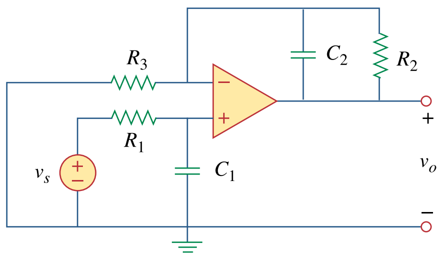
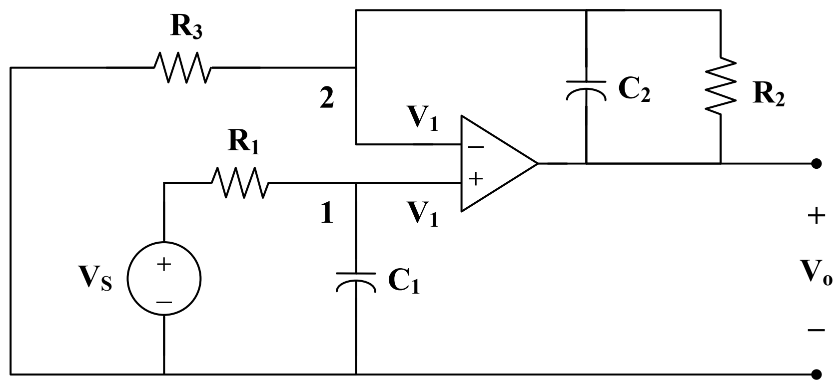


Figure 10.120
For Prob. 10.77.

Chapter 10, Solution 77.

Consider the circuit below.



At node 1,

$$\frac{\mathbf{V}_s - \mathbf{V}_1}{R_1} = j\omega C\,\mathbf{V}_1$$

$$\mathbf{V}_s = (1 + j\omega R_1 C_1)\,\mathbf{V}_1 \qquad (1)$$

At node 2,

$$\frac{0 - \mathbf{V}_1}{R_3} = \frac{\mathbf{V}_1 - \mathbf{V}_o}{R_2} + j\omega C_2\,(\mathbf{V}_1 - \mathbf{V}_o)$$

$$\mathbf{V}_1 = (\mathbf{V}_o - \mathbf{V}_1)\left(\frac{R_3}{R_2} + j\omega C_2 R_3\right)$$

$$\mathbf{V}_o = \left(1 + \frac{1}{(R_3/R_2) + j\omega C_2 R_3}\right)\mathbf{V}_1 \qquad (2)$$

From (1) and (2),

$$\mathbf{V}_o = \frac{\mathbf{V}_s}{1 + j\omega R_1 C_1}\left(1 + \frac{R_2}{R_3 + j\omega C_2 R_2 R_3}\right)$$

$$\frac{\mathbf{V}_o}{\mathbf{V}_s} = \underline{\frac{\mathbf{R_2 + R_3 + j\omega C_2 R_2 R_3}}{\mathbf{(1 + j\omega R_1 C_1)(R_3 + j\omega C_2 R_2 R_3)}}}$$

Chapter 10, Problem 78.

Determine $v_o(t)$ in the op amp circuit in Fig. 10.121 below.

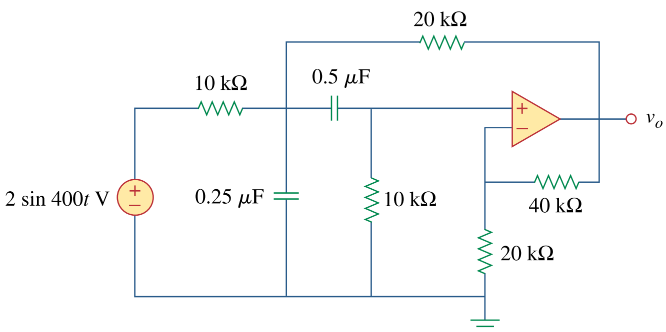


Figure 10.121
For Prob. 10.78.

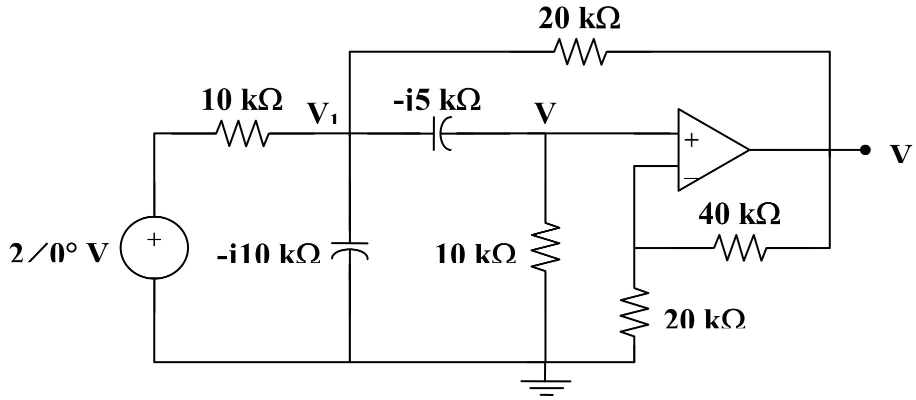
Chapter 10, Solution 78.

$$2\sin(400t) \longrightarrow 2\angle 0°, \quad \omega = 400$$

$$0.5\ \mu F \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(400)(0.5\times 10^{-6})} = -j5\ k\Omega$$

$$0.25\ \mu F \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(400)(0.25\times 10^{-6})} = -j10\ k\Omega$$

Consider the circuit as shown below.



At node 1,

$$\frac{2 - \mathbf{V}_1}{10} = \frac{\mathbf{V}_1}{-j10} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{-j5} + \frac{\mathbf{V}_1 - \mathbf{V}_o}{20}$$

$$4 = (3 + j6)\,\mathbf{V}_1 - j4\,\mathbf{V}_2 - \mathbf{V}_o \tag{1}$$

At node 2,

$$\frac{\mathbf{V}_1 - \mathbf{V}_2}{-j5} = \frac{\mathbf{V}_2}{10}$$

$$\mathbf{V}_1 = (1 - j0.5)\,\mathbf{V}_2 \tag{2}$$

But

$$\mathbf{V}_2 = \frac{20}{20 + 40}\mathbf{V}_o = \frac{1}{3}\mathbf{V}_o \tag{3}$$

From (2) and (3),

$$\mathbf{V}_1 = \frac{1}{3}\cdot(1 - j0.5)\,\mathbf{V}_o \tag{4}$$

Substituting (3) and (4) into (1) gives

$$4 = (3 + j6)\cdot\frac{1}{3}\cdot(1 - j0.5)\,\mathbf{V}_o - j\frac{4}{3}\mathbf{V}_o - \mathbf{V}_o = \left(1 + j\frac{1}{6}\right)\mathbf{V}_o$$

$$\mathbf{V}_o = \frac{24}{6 + j} = 3.945\angle -9.46°$$

Therefore,

$$v_o(t) = \underline{\mathbf{3.945\ sin(400t - 9.46°)\ V}}$$

Chapter 10, Problem 79.

For the op amp circuit in Fig. 10.122, obtain $v_o(t)$.

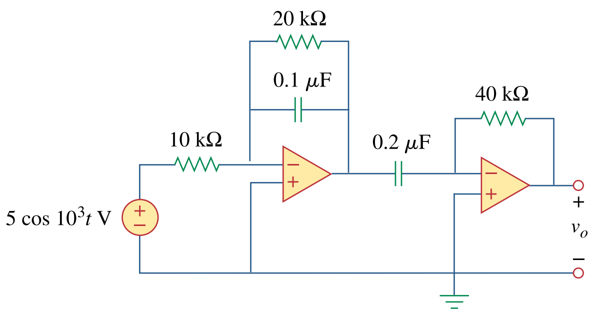


Figure 10.122
For Prob. 10.79.

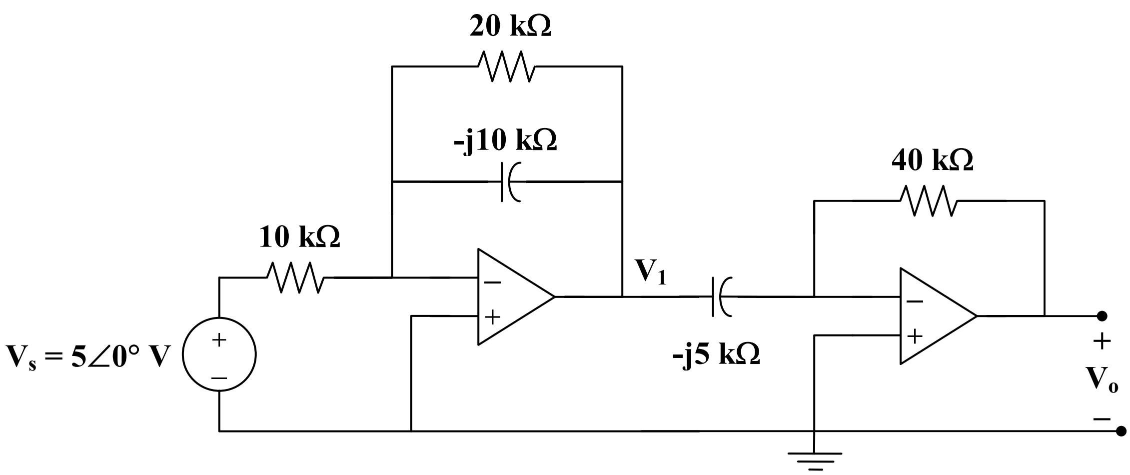
Chapter 10, Solution 79.

$$5\cos(1000t) \longrightarrow 5\angle 0°, \quad \omega = 1000$$

$$0.1\ \mu\text{F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(1000)(0.1\times 10^{-6})} = \text{-}j10\ \text{k}\Omega$$

$$0.2\ \mu\text{F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(1000)(0.2\times 10^{-6})} = \text{-}j5\ \text{k}\Omega$$

Consider the circuit shown below.



Since each stage is an inverter, we apply $\mathbf{V}_o = \frac{-\mathbf{Z}_f}{\mathbf{Z}_i}\mathbf{V}_i$ to each stage.

$$\mathbf{V}_o = \frac{-40}{-j5}\mathbf{V}_1 \tag{1}$$

and

$$\mathbf{V}_1 = \frac{-20\,\|\,(-j10)}{10}\mathbf{V}_s \tag{2}$$

From (1) and (2),

$$\mathbf{V}_o = \left(\frac{-j8}{10}\right)\left(\frac{-(20)(-j10)}{20-j10}\right)5\angle 0°$$

$$\mathbf{V}_o = 16(2+j) = 35.78\angle 26.56°$$

Therefore, $v_o(t) =$ **35.78 cos(1000t + 26.56°) V**

Chapter 10, Problem 80.

Obtain $v_o(t)$ for the op amp circuit in Fig. 10.123 if $v_s = 4\cos(1000t - 60°)$ V.

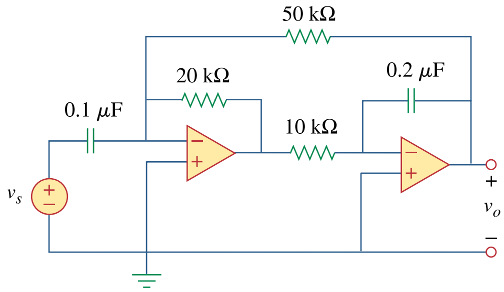


Figure 10.123
For Prob. 10.80.

Chapter 10, Solution 80.

$$4\cos(1000t - 60°) \longrightarrow 4\angle -60°, \quad \omega = 1000$$

$$0.1\ \mu\text{F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(1000)(0.1\times 10^{-6})} = -j10\ \text{k}\Omega$$

$$0.2\ \mu\text{F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(1000)(0.2\times 10^{-6})} = -j5\ \text{k}\Omega$$

The two stages are inverters so that

$$\mathbf{V}_o = \left(\frac{20}{-j10}\cdot(4\angle -60°) + \frac{20}{50}\mathbf{V}_o\right)\left(\frac{-j5}{10}\right)$$

$$= \frac{-j}{2}\cdot(j2)\cdot(4\angle -60°) + \frac{-j}{2}\cdot\frac{2}{5}\mathbf{V}_o$$

$$(1 + j/5)\,\mathbf{V}_o = 4\angle -60°$$

$$\mathbf{V}_o = \frac{4\angle -60°}{1 + j/5} = 3.922\angle -71.31°$$

Therefore, $v_o(t) =$ **3.922 cos(1000t – 71.31°) V**

Chapter 10, Problem 81.

Use *PSpice* to determine $\mathbf{V}_o$ in the circuit of Fig. 10.124. Assume $\omega = 1$ rad/s.

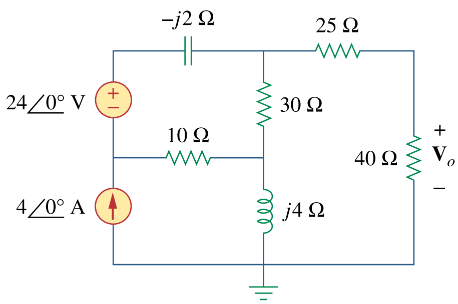


Figure 10.124
For Prob. 10.81.

Chapter 10, Problem 89.

The op amp circuit in Fig. 10.131 is called an *inductance simulator*. Show that the input impedance is given by

$$\mathbf{Z}_{\text{in}} = \frac{\mathbf{V}_{\text{in}}}{\mathbf{I}_{\text{in}}} = j\omega L_{\text{eq}}$$

where

$$L_{\text{eq}} = \frac{R_1 R_3 R_4}{R_2} C$$

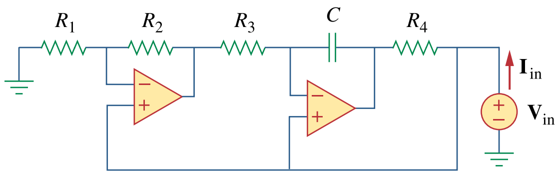
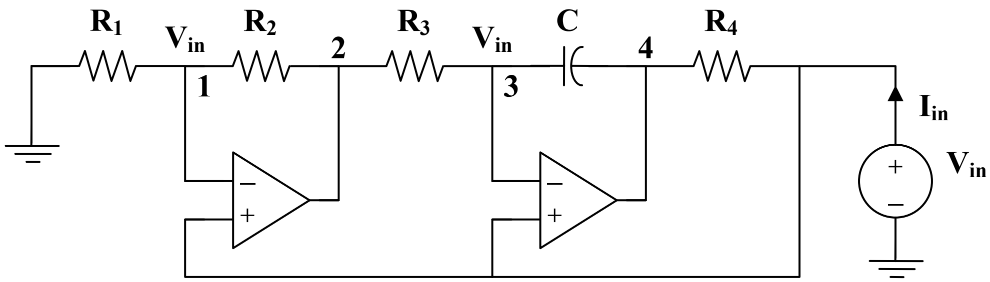


Figure 10.131
For Prob. 10.89.

Chapter 10, Solution 89.

Consider the circuit below.



At node 1,

$$\frac{0-\mathbf{V}_{in}}{R_1} = \frac{\mathbf{V}_{in}-\mathbf{V}_2}{R_2}$$

$$-\mathbf{V}_{in} + \mathbf{V}_2 = \frac{R_2}{R_1}\mathbf{V}_{in} \qquad (1)$$

At node 3,

$$\frac{\mathbf{V}_2-\mathbf{V}_{in}}{R_3} = \frac{\mathbf{V}_{in}-\mathbf{V}_4}{1/j\omega C}$$

$$-\mathbf{V}_{in} + \mathbf{V}_4 = \frac{\mathbf{V}_{in}-\mathbf{V}_2}{j\omega C R_3} \qquad (2)$$

From (1) and (2),

$$-\mathbf{V}_{in} + \mathbf{V}_4 = \frac{-R_2}{j\omega C R_3 R_1}\mathbf{V}_{in}$$

Thus,

$$\mathbf{I}_{in} = \frac{\mathbf{V}_{in}-\mathbf{V}_4}{R_4} = \frac{R_2}{j\omega C R_3 R_1 R_4}\mathbf{V}_{in}$$

$$\mathbf{Z}_{in} = \frac{\mathbf{V}_{in}}{\mathbf{I}_{in}} = \frac{j\omega C R_1 R_3 R_4}{R_2} = j\omega L_{eq}$$

where $\underline{\mathbf{L_{eq} = \frac{R_1 R_3 R_4 C}{R_2}}}$

Chapter 10, Problem 90.

Figure 10.132 shows a Wien-bridge network. Show that the frequency at which the phase shift between the input and output signals is zero is $f = \frac{1}{2}\pi RC$, and that the necessary gain is $\mathbf{A}_v = \mathbf{V}_o/\mathbf{V}_i = 3$ at that frequency.

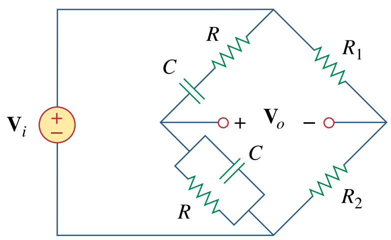


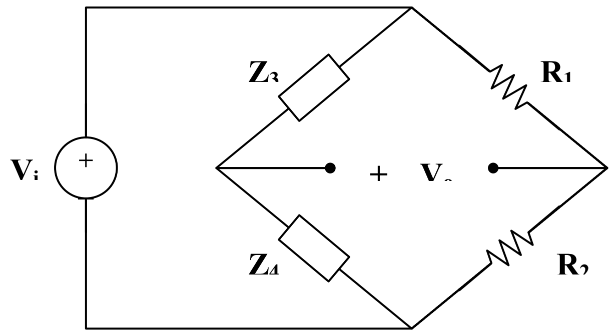
Figure 10.132
For Prob. 10.90.

Chapter 10, Solution 90.

Let $$\mathbf{Z}_4 = R \parallel \frac{1}{j\omega C} = \frac{R}{1 + j\omega RC}$$

$$\mathbf{Z}_3 = R + \frac{1}{j\omega C} = \frac{1 + j\omega RC}{j\omega C}$$

Consider the circuit shown below.



$$\mathbf{V}_o = \frac{\mathbf{Z}_4}{\mathbf{Z}_3 + \mathbf{Z}_4}\mathbf{V}_i - \frac{R_2}{R_1 + R_2}\mathbf{V}_i$$

$$\frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{\dfrac{R}{1 + j\omega C}}{\dfrac{R}{1 + j\omega C} + \dfrac{1 + j\omega RC}{j\omega C}} - \frac{R_2}{R_1 + R_2}$$

$$= \frac{j\omega RC}{j\omega RC + (1 + j\omega RC)^2} - \frac{R_2}{R_1 + R_2}$$

$$\frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{j\omega RC}{1 - \omega^2 R^2 C^2 + j3\omega RC} - \frac{R_2}{R_1 + R_2}$$

For $\mathbf{V}_o$ and $\mathbf{V}_i$ to be in phase, $\dfrac{\mathbf{V}_o}{\mathbf{V}_i}$ must be purely real. This happens when

$$1 - \omega^2 R^2 C^2 = 0$$

$$\omega = \frac{1}{RC} = 2\pi f$$

or $$f = \frac{1}{2\pi RC}$$

At this frequency, $$\mathbf{A}_v = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{1}{3} - \frac{R_2}{R_1 + R_2}$$

Chapter 10, Problem 91.

Consider the oscillator in Fig. 10.133.
(a) Determine the oscillation frequency.
(b) Obtain the minimum value of *R* for which oscillation takes place.

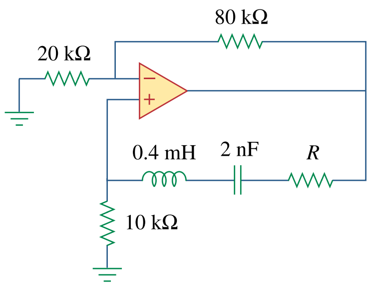


Figure 10.133
For Prob. 10.91.

Chapter 10, Solution 91.

(a) Let $\mathbf{V}_2 =$ voltage at the noninverting terminal of the op amp

$\mathbf{V}_o =$ output voltage of the op amp

$\mathbf{Z}_p = 10\ \text{k}\Omega = R_o$

$$\mathbf{Z}_s = R + j\omega L + \frac{1}{j\omega C}$$

As in Section 10.9,

$$\frac{\mathbf{V}_2}{\mathbf{V}_o} = \frac{\mathbf{Z}_p}{\mathbf{Z}_s + \mathbf{Z}_p} = \frac{R_o}{R + R_o + j\omega L - \dfrac{j}{\omega C}}$$

$$\frac{\mathbf{V}_2}{\mathbf{V}_o} = \frac{\omega C R_o}{\omega C (R + R_o) + j(\omega^2 LC - 1)}$$

For this to be purely real,

$$\omega_o^2 LC - 1 = 0 \quad \longrightarrow \quad \omega_o = \frac{1}{\sqrt{LC}}$$

$$f_o = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(0.4\times 10^{-3})(2\times 10^{-9})}}$$

$f_o =$ **180 kHz**

(b) At oscillation,

$$\frac{\mathbf{V}_2}{\mathbf{V}_o} = \frac{\omega_o C R_o}{\omega_o C (R + R_o)} = \frac{R_o}{R + R_o}$$

This must be compensated for by

$$\mathbf{A}_v = \frac{\mathbf{V}_o}{\mathbf{V}_2} = 1 + \frac{80}{20} = 5$$

$$\frac{R_o}{R + R_o} = \frac{1}{5} \quad \longrightarrow \quad R = 4R_o = \mathbf{40\ k\Omega}$$

Chapter 10, Problem 92.

The oscillator circuit in Fig. 10.134 uses an ideal op amp.
(a) Calculate the minimum value of R_o that will cause oscillation to occur.
(b) Find the frequency of oscillation.

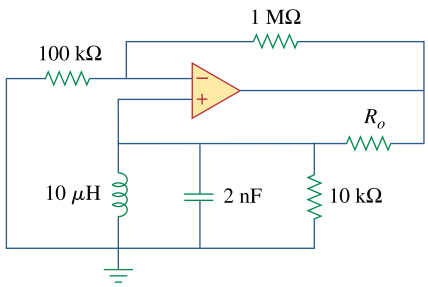


Figure 10.134
For Prob. 10.92.

Chapter 10, Solution 92.

Let $\mathbf{V}_2 =$ voltage at the noninverting terminal of the op amp
$\mathbf{V}_o =$ output voltage of the op amp
$\mathbf{Z}_s = R_o$

$$\mathbf{Z}_p = j\omega L \,||\, \frac{1}{j\omega C} \,||\, R = \frac{1}{\frac{1}{R} + j\omega C + \frac{1}{j\omega L}} = \frac{\omega RL}{\omega L + jR(\omega^2 LC - 1)}$$

As in Section 10.9,

$$\frac{\mathbf{V}_2}{\mathbf{V}_o} = \frac{\mathbf{Z}_p}{\mathbf{Z}_s + \mathbf{Z}_p} = \frac{\frac{\omega RL}{\omega L + jR(\omega^2 LC - 1)}}{R_o + \frac{\omega RL}{\omega L + jR(\omega^2 LC - 1)}}$$

$$\frac{\mathbf{V}_2}{\mathbf{V}_o} = \frac{\omega RL}{\omega RL + \omega R_o L + jR_o R\,(\omega^2 LC - 1)}$$

For this to be purely real,

$$\omega_o^2 LC = 1 \longrightarrow f_o = \frac{1}{2\pi\sqrt{LC}}$$

(a) At $\omega = \omega_o$,

$$\frac{\mathbf{V}_2}{\mathbf{V}_o} = \frac{\omega_o RL}{\omega_o RL + \omega_o R_o L} = \frac{R}{R + R_o}$$

This must be compensated for by

$$\mathbf{A}_v = \frac{\mathbf{V}_o}{\mathbf{V}_2} = 1 + \frac{R_f}{R_o} = 1 + \frac{1000k}{100k} = 11$$

Hence,

$$\frac{R}{R + R_o} = \frac{1}{11} \longrightarrow R_o = 10R = \underline{\mathbf{100\ k\Omega}}$$

(b)

$$f_o = \frac{1}{2\pi\sqrt{(10\times10^{-6})(2\times10^{-9})}}$$

$$f_o = \underline{\mathbf{1.125\ MHz}}$$

Chapter 10, Problem 93.

Figure 10.135 shows a *Colpitts oscillator*. Show that the oscillation frequency is

$$f_o = \frac{1}{2\pi\sqrt{LC_T}}$$

where $C_T = C_1C_2/(C_1 + C_2)$. Assume $R_i >> X_{C_2}$

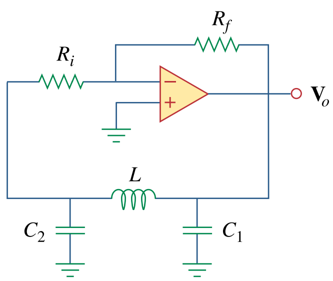


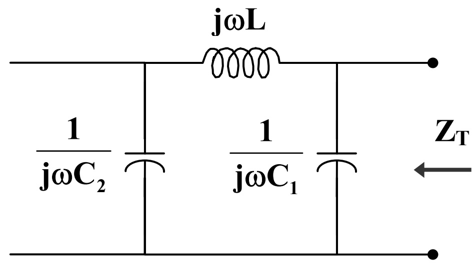
Figure 10.135

A Colpitts oscillator; for Prob. 10.93.

(*Hint:* Set the imaginary part of the impedance in the feedback circuit equal to zero.)

Chapter 10, Solution 93.

As shown below, the impedance of the feedback is



$$\mathbf{Z}_T = \frac{1}{j\omega C_1} \parallel \left(j\omega L + \frac{1}{j\omega C_2} \right)$$

$$\mathbf{Z}_T = \frac{\frac{-j}{\omega C_1}\left(j\omega L + \frac{-j}{\omega C_2} \right)}{\frac{-j}{\omega C_1} + j\omega L + \frac{-j}{\omega C_2}} = \frac{\frac{1}{\omega} - \omega L C_2}{j(C_1 + C_2 - \omega^2 L C_1 C_2)}$$

In order for $\mathbf{Z}_T$ to be real, the imaginary term must be zero; i.e.

$$C_1 + C_2 - \omega_o^2 L C_1 C_2 = 0$$

$$\omega_o^2 = \frac{C_1 + C_2}{L C_1 C_2} = \frac{1}{L C_T}$$

$$\underline{\mathbf{f_o = \frac{1}{2\pi\sqrt{LC_T}}}}$$

Chapter 10, Problem 94.

Design a Colpitts oscillator that will operate at 50 kHz.

Chapter 10, Solution 94.

If we select $C_1 = C_2 = 20 \text{ nF}$

$$C_T = \frac{C_1 C_2}{C_1 + C_2} = \frac{C_1}{2} = 10 \text{ nF}$$

Since $f_o = \dfrac{1}{2\pi\sqrt{LC_T}}$,

$$L = \frac{1}{(2\pi f)^2 C_T} = \frac{1}{(4\pi^2)(2500\times 10^6)(10\times 10^{-9})} = 10.13 \text{ mH}$$

$$X_c = \frac{1}{\omega C_2} = \frac{1}{(2\pi)(50\times 10^3)(20\times 10^{-9})} = 159\ \Omega$$

We may select $R_i = 20\ k\Omega$ and $R_f \geq R_i$, say $R_f = 20\ k\Omega$.

Thus,

$C_1 = C_2 =$ **20 nF**, $\quad L =$ **10.13 mH** $\quad R_f = R_i =$ **20 kΩ**

Chapter 10, Problem 95.
Figure 10.136 shows a *Hartley oscillator*. Show that the frequency of oscillation is

$$f_o = \frac{1}{2\pi\sqrt{C(L_1 + L_2)}}$$

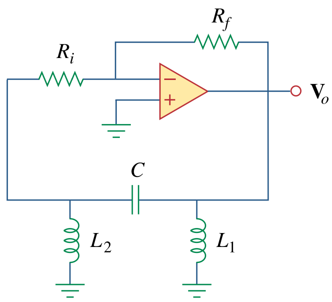
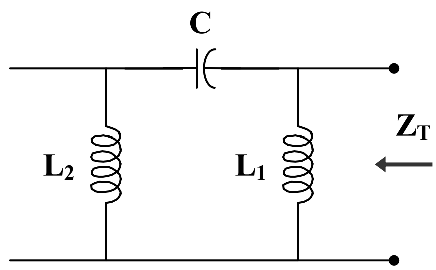


Figure 10.136
A Hartley oscillator; For Prob. 10.95.

Chapter 10, Solution 95.
First, we find the feedback impedance.



$$\mathbf{Z}_T = j\omega L_1 \,\|\, \left(j\omega L_2 + \frac{1}{j\omega C}\right)$$

$$\mathbf{Z}_T = \frac{j\omega L_1\left(j\omega L_2 - \dfrac{j}{\omega C}\right)}{j\omega L_1 + j\omega L_2 - \dfrac{j}{\omega C}} = \frac{\omega^2 L_1 C(1 - \omega L_2)}{j(\omega^2 C(L_1 + L_2) - 1)}$$

In order for $\mathbf{Z}_T$ to be real, the imaginary term must be zero; i.e.

$$\omega_o^2 C(L_1 + L_2) - 1 = 0$$

$$\omega_o = 2\pi f_o = \frac{1}{C(L_1 + L_2)}$$

$$\mathbf{f_o = \frac{1}{2\pi\sqrt{C(L_1 + L_2)}}}$$

Chapter 10, Problem 96.

Refer to the oscillator in Fig. 10.137.

(a) Show that

$$\frac{\mathbf{V}_2}{\mathbf{V}_o} = \frac{1}{3 + j(\omega L / R - R / \omega L)}$$

(b) Determine the oscillation frequency f_o.

(c) Obtain the relationship between R_1 and R_2 in order for oscillation to occur.

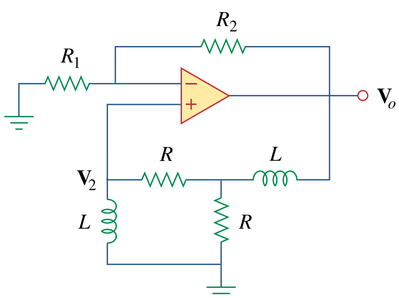
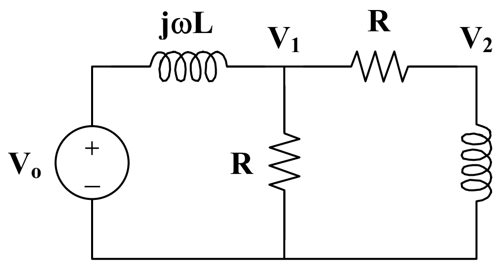


Figure 10.137
For Prob. 10.96.

Chapter 10, Solution 96.

(a) Consider the feedback portion of the circuit, as shown below.



$$\mathbf{V_2} = \frac{j\omega L}{R + j\omega L}\mathbf{V_1} \longrightarrow \mathbf{V}_1 = \frac{R + j\omega L}{j\omega L}\mathbf{V}_2 \qquad (1)$$

Applying KCL at node 1,

$$\frac{\mathbf{V}_o - \mathbf{V}_1}{j\omega L} = \frac{\mathbf{V}_1}{R} + \frac{\mathbf{V}_1}{R + j\omega L}$$

$$\mathbf{V}_o - \mathbf{V}_1 = j\omega L\,\mathbf{V}_1\left(\frac{1}{R} + \frac{1}{R + j\omega L}\right)$$

$$\mathbf{V}_o = \mathbf{V}_1\left(1 + \frac{j2\omega RL - \omega^2 L^2}{R\,(R + j\omega L)}\right)$$

(2)

From (1) and (2),

$$\mathbf{V}_o = \left(\frac{R + j\omega L}{j\omega L}\right)\left(1 + \frac{j2\omega RL - \omega^2 L^2}{R\,(R + j\omega L)}\right)\mathbf{V}_2$$

$$\frac{\mathbf{V}_o}{\mathbf{V}_2} = \frac{R^2 + j\omega RL + j2\omega RL - \omega^2 L^2}{j\omega RL}$$

$$\frac{\mathbf{V}_2}{\mathbf{V}_o} = \frac{1}{3 + \dfrac{R^2 - \omega^2 L^2}{j\omega RL}}$$

$$\frac{\mathbf{V}_2}{\mathbf{V}_o} = \underline{\frac{\mathbf{1}}{\mathbf{3 + j(\omega L/R - R/\omega L)}}}$$

(b) Since the ratio $\frac{\mathbf{V}_2}{\mathbf{V}_o}$ must be real,

$$\frac{\omega_o L}{R} - \frac{R}{\omega_o L} = 0$$

$$\omega_o L = \frac{R^2}{\omega_o L}$$

$$\omega_o = 2\pi f_o = \frac{R}{L}$$

$$\underline{\mathbf{f_o = \frac{R}{2\pi L}}}$$

(c) When $\omega = \omega_o$

$$\frac{\mathbf{V}_2}{\mathbf{V}_o} = \frac{1}{3}$$

This must be compensated for by $\mathbf{A}_v = 3$. But

$$\mathbf{A}_v = 1 + \frac{R_2}{R_1} = 3$$

$$\underline{\mathbf{R_2 = 2R_1}}$$